Charting the Origin of Hadron Masses

Craig Roberts, Physics Division

Collaborators: 2013-Present

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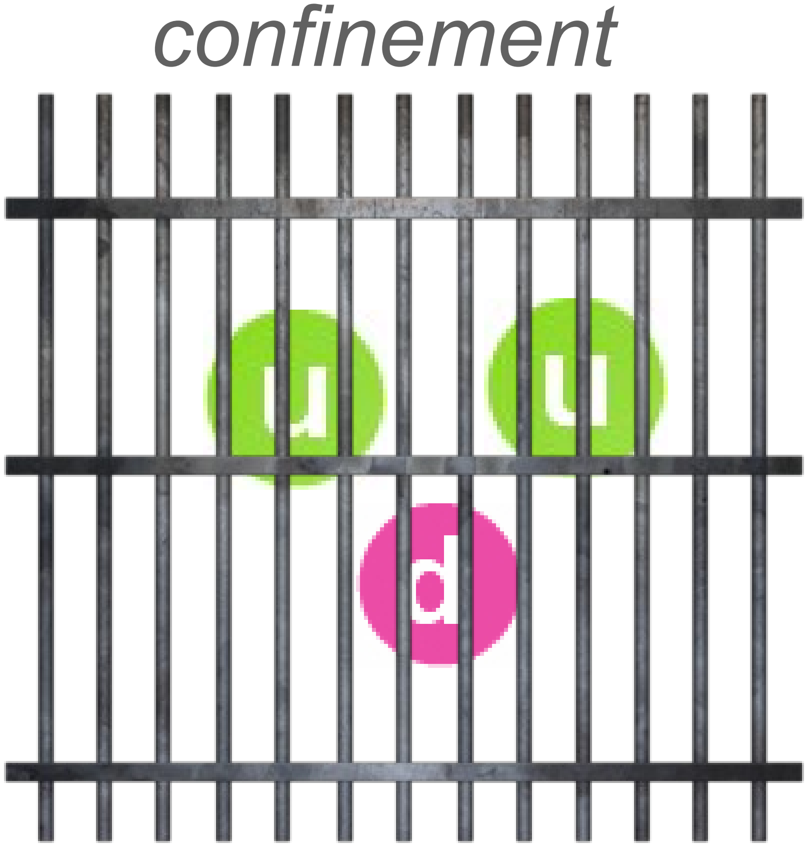
2013: Higgs and Englert

- “The Higgs boson is often said to give mass to everything. However, that is wrong. It only gives mass to some very simple particles, accounting for only one or two percent of the mass of more complex things like atoms, molecules and everyday objects, from your mobile phone to your pet llama.”
- *“The vast majority of mass comes from the energy needed to hold quarks together inside nuclei.”*



What & where is mass?

$$\mathcal{L}_{\rm QCD} = \bar{\psi}_i \left(i(\gamma^\mu D_\mu)_{ij} \qquad\qquad \right) \psi_j - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a$$

Whence Mass?

- Classical chromdynamics … non-Abelian local gauge theory
- Remove the current mass … there’s no energy scale left
- *No dynamics in a scale-invariant theory*; only kinematics … the theory looks the same at all length-scales … there can be no clumps of anything … *hence bound-states are impossible*.
- *Our Universe can’t exist*
- *Higgs boson doesn’t solve this problem* … normal matter is constituted from light-quarks & the mass of protons and neutrons, the kernels of all visible matter, are 100-times larger than anything the Higgs can produce
- *Where did it all begin?*

… becomes … Where did it all come from?

Whence Mass?

- Poincaré invariance entails that the Energy-Momentum Tensor is divergence-free, *i.e*. it defines a conserved current:

$$\partial_\mu T_{\mu\nu} = 0$$

$T\mu\nu$ can *always* be made symmetric

- Noether current global scale transformation:

  $x \longrightarrow e-\sigma\, x$

  is the dilation current: $D\mu\nu = T\mu\nu\, x\nu$

- In a scale invariant theory, the dilation current is conserved

$$\partial_\mu \mathcal{D}_\mu = 0 = [\partial_\mu T_{\mu\nu}] x_\nu + T_{\mu\nu}\delta_{\mu\nu}$$
$$= T_{\mu\mu}\,,$$

- Conseque
  the *energy-momentum tensor must be traceless*.

Trace Anomaly

- Classical chromodynamics is plainly meaningless … must be quantised
- Regularisation and renormalisation of (ultraviolet) divergences introduces a mass-scale

… *dimensional transmutation*: mass-dimensionless quantities become dependent on a mass-scale, ζ

- $\alpha \to \alpha(\zeta)$ in QCD's (massless) Lagrangian density, L(m=0)

Under a scale transformation $\zeta \to e^{\sigma}\zeta$, then $\alpha \to \sigma\, \alpha\beta(\alpha)$

$$L \to \sigma\, \alpha\beta(\alpha)\, dL/d\alpha$$

Trace anomaly

$$\Rightarrow \partial_\mu D_\mu = \delta L/\delta\sigma = \alpha\beta(\alpha)\, dL/d\alpha \boxed{= \beta(\alpha)\, \tfrac{1}{4} G_{\mu\nu} G_{\mu\nu} = T_{\rho\rho}} =: \Theta_0$$

- Straightforward, nonperturbative derivation, without need for diagrammatic analysis …

quantisation of renormalisable four-dimensional theory forces nonzero value for trace of energy-momentum tensor

Where is the mass?

$$T_{\mu\mu} = \frac{1}{4}\beta(\alpha(\zeta))G^a_{\mu\nu}G^a_{\mu\nu}$$

Trace Anomaly

- Knowing that a trace anomaly exists does not deliver a great deal … indicates only that a mass-scale exists
- Can one compute and/or understand the magnitude of that scale?
- One can certainly *measure* the magnitude … consider proton:

$$\langle p(P)|T_{\mu\nu}|p(P)\rangle = -P_\mu P_\nu$$

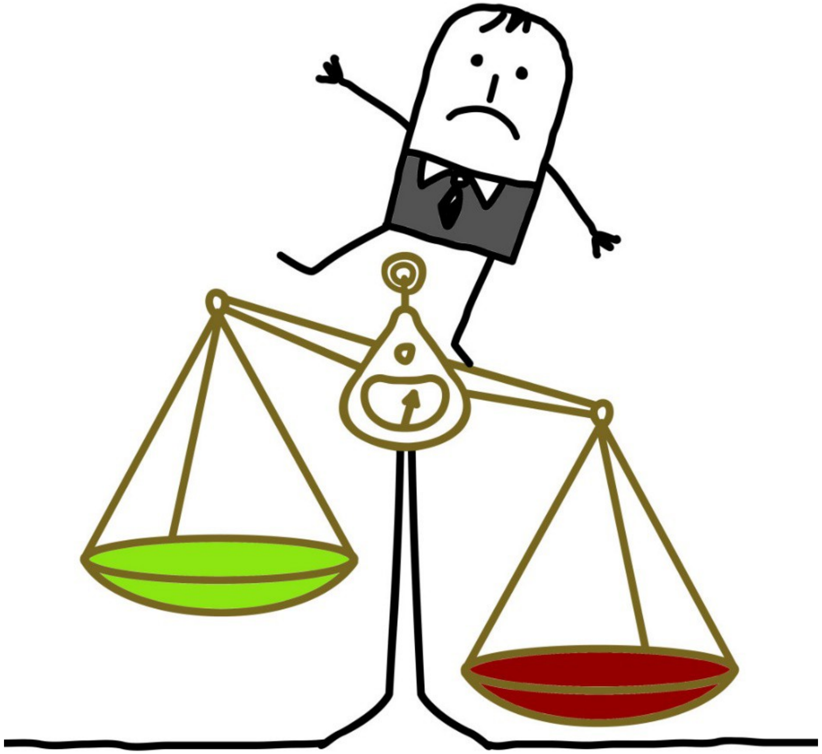
$$\langle p(P)|T_{\mu\mu}|p(P)\rangle = -P^2 = m_p^2$$

- In t
anc

$$= \langle p(P)|\Theta_0|p(P)\rangle$$

… In QCD, *Θ0* measures the strength of gluon self-interactions

… so, from one perspective, *mp* is completely generated by glue.



On the other hand …

$$T_{\mu\mu} = \frac{1}{4}\beta(\alpha(\zeta))G^a_{\mu\nu}G^a_{\mu\nu}$$

Trace Anomaly

- In the chiral limit

$$\langle\pi(q)|T_{\mu\nu}|\pi(q)\rangle = -q_\mu q_\nu \qquad \langle\pi(q)|\Theta_0|\pi(q)\rangle = 0$$

- Does this mean that the scale anomaly vanishes trivially in the pion state, *i.e*. gluons contribute nothing to the pion mass?
- That is a difficult way to obtain “zero”

- Easier, perhaps, to imagine that “zero” owes to cancellations between different operator-component contributions to $\Theta 0$.
- Of course, such precise cancellation should not be an accident.

It could only arise naturally because of some symmetry and/or symmetry-breaking pattern.

$$T_{\mu\mu} = \frac{1}{4}\beta(\alpha(\zeta))G^a_{\mu\nu}G^a_{\mu\nu}$$

Trace Anomaly

- In the chiral limit

$$\langle \pi(q)|T_{\mu\nu}|\pi(q)\rangle = -q_\mu q_\nu \qquad \langle \pi(q)|\Theta_0|\pi(q)\rangle = 0$$

- *No statement of the question*
  "Whence the proton's mass?"
  is complete without the additional clause
  "Whence the absence of a pion mass?"

Whence "1" and yet "0"

$$\langle p(P)|\Theta_0|p(P)\rangle = m_p^2\,, \quad \langle \pi(q)|\Theta_0|\pi(q)\rangle = 0$$

- Both statements are Poincaré invariant
- In connection with any bound-state, the only things that any two observers can certainly agree upon are the eigenvalues of the two Casimir operators of the Poincaré group evaluated in that state:
  - $M2 \rightarrow m2$ & $W2 \rightarrow$ m2 $j(j+1)$

    … the mass and total spin
- No decomposition of these quantities into separate contributions from constituents can ever be Poincaré-invariant or scale-invariant
  - This fact lies at the heart of the so-called "spin-crisis", which could therefore have been avoided

Whence "1" and yet "0"

$$\langle p(P)|\Theta_0|p(P)\rangle = m_p^2\,, \quad \langle \pi(q)|\Theta_0|\pi(q)\rangle = 0$$

- Even using light-front quantisation, both the *natures* of and *contributions* from constituents changes with resolving scale, ζ
- Can it be sensible to attempt an expression of these trace anomaly statements in a particular frame, *e.g*. a hadron's rest-frame?
- *Naturally not*

  … a massless particle doesn't have a rest frame

$$\mathcal{L}_{\text{QCD}} = \bar{\psi}_i \left(i(\gamma^\mu D_\mu)_{ij} \qquad\qquad \right) \psi_j - \frac{1}{4} G^a_{\mu\nu} G_a^{\mu\nu}$$

Whence?

- Classical chromodynamics … non-Abelian local gauge theory
- Local gauge invariance; but there is no confinement without a mass-scale
  - Three quarks can still be colour-singlet
  - Colour rotations will keep them colour singlets
  - But they need have no proximity to one another
    … proximity is meaningless in a scale-invariant theory
- Whence mass … equivalent to whence a mass-scale … equivalent to whence a confinement scale
- *Understanding the origin and absence of mass in QCD is quite likely inseparable from the task of understanding confinement. Existence alone of a scale anomaly answers neither question*

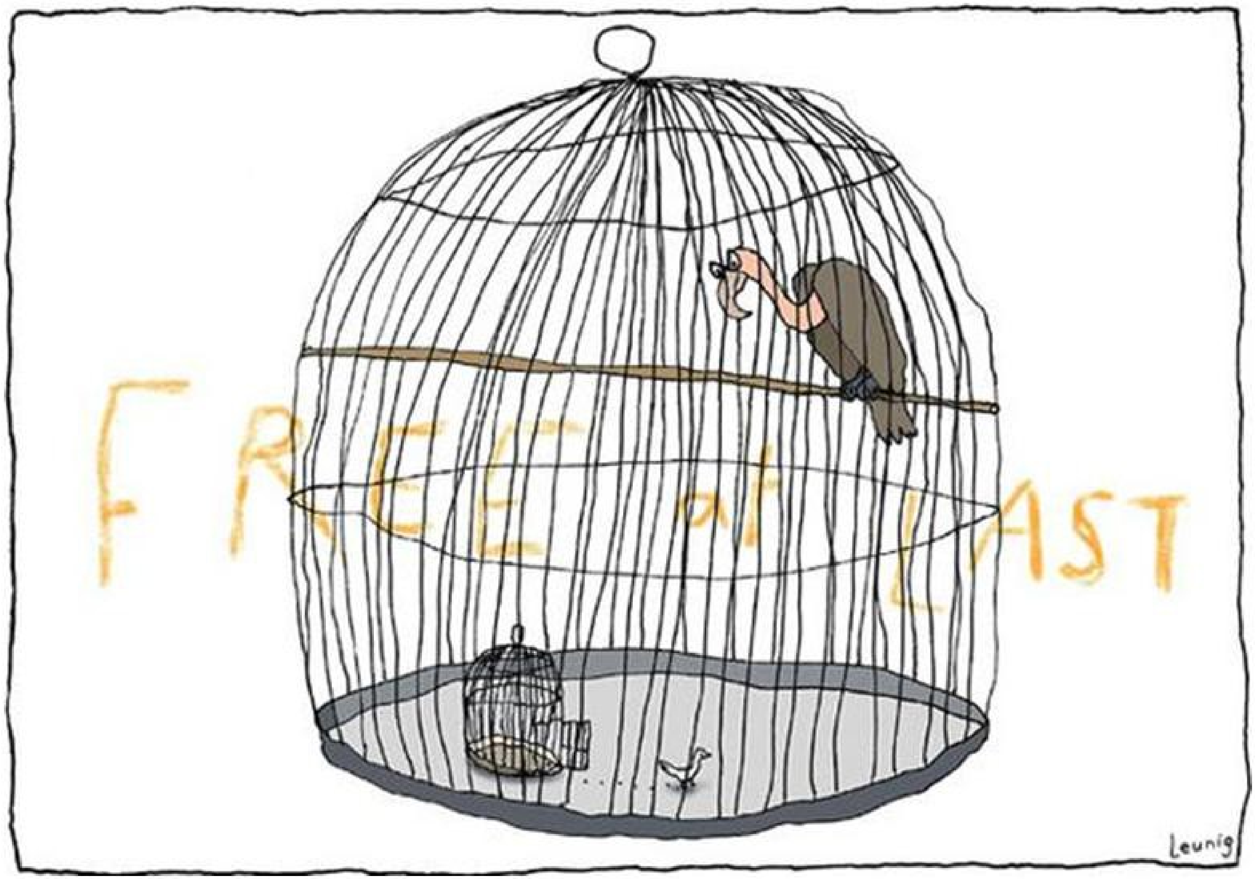
A New Era for hadro-particle physics

Overarching Science Challenges for the coming decade

- What is origin of mass in our Universe?
- What is the nature of confinement in real (dynamical-quarks) QCD?
- How are they connected?
- How can any
  - answers,
  - conjectures
  - and/or conclusions

  be empirically verified?

Physics is an Empirical Science



What is Confinement?

Confinement?

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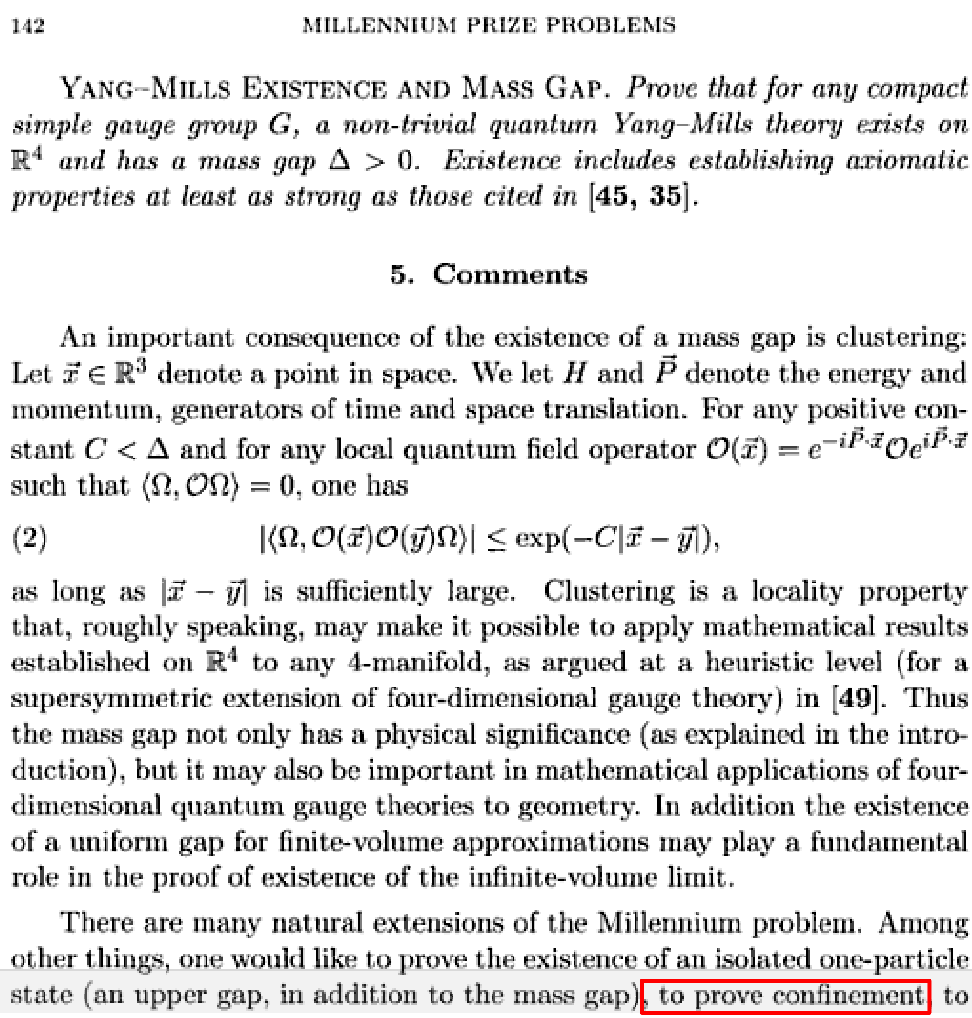
- ***Millennium prize of $1,000,000 for proving that SUc(3) gauge theory is mathematically well-defined, which will necessarily prove or disprove the confinement conjecture***



142 MILLENNIUM PRIZE PROBLEMS

Yang–Mills Existence and Mass Gap. *Prove that for any compact simple gauge group G, a non-trivial quantum Yang–Mills theory exists on $\mathbb{R}^4$ and has a mass gap $\Delta > 0$. Existence includes establishing axiomatic properties at least as strong as those cited in* [**45, 35**].

5. Comments

An important consequence of the existence of a mass gap is clustering: Let $\vec{x} \in \mathbb{R}^3$ denote a point in space. We let H and $\vec{P}$ denote the energy and momentum, generators of time and space translation. For any positive constant $C < \Delta$ and for any local quantum field operator $\mathcal{O}(\vec{x}) = e^{-i\vec{P}\cdot\vec{x}}\mathcal{O}e^{i\vec{P}\cdot\vec{x}}$ such that $\langle\Omega, \mathcal{O}\Omega\rangle = 0$, one has

$$|\langle\Omega, \mathcal{O}(\vec{x})\mathcal{O}(\vec{y})\Omega\rangle| \leq \exp(-C|\vec{x} - \vec{y}|), \tag{2}$$

as long as $|\vec{x} - \vec{y}|$ is sufficiently large. Clustering is a locality property that, roughly speaking, may make it possible to apply mathematical results established on $\mathbb{R}^4$ to any 4-manifold, as argued at a heuristic level (for a supersymmetric extension of four-dimensional gauge theory) in [**49**]. Thus the mass gap not only has a physical significance (as explained in the introduction), but it may also be important in mathematical applications of four-dimensional quantum gauge theories to geometry. In addition the existence of a uniform gap for finite-volume approximations may play a fundamental role in the proof of existence of the infinite-volume limit.

There are many natural extensions of the Millennium problem. Among other things, one would like to prove the existence of an isolated one-particle state (an upper gap, in addition to the mass gap), to prove confinement, to

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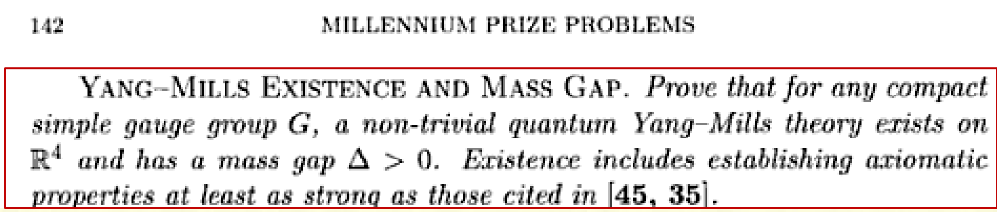

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Confinement?

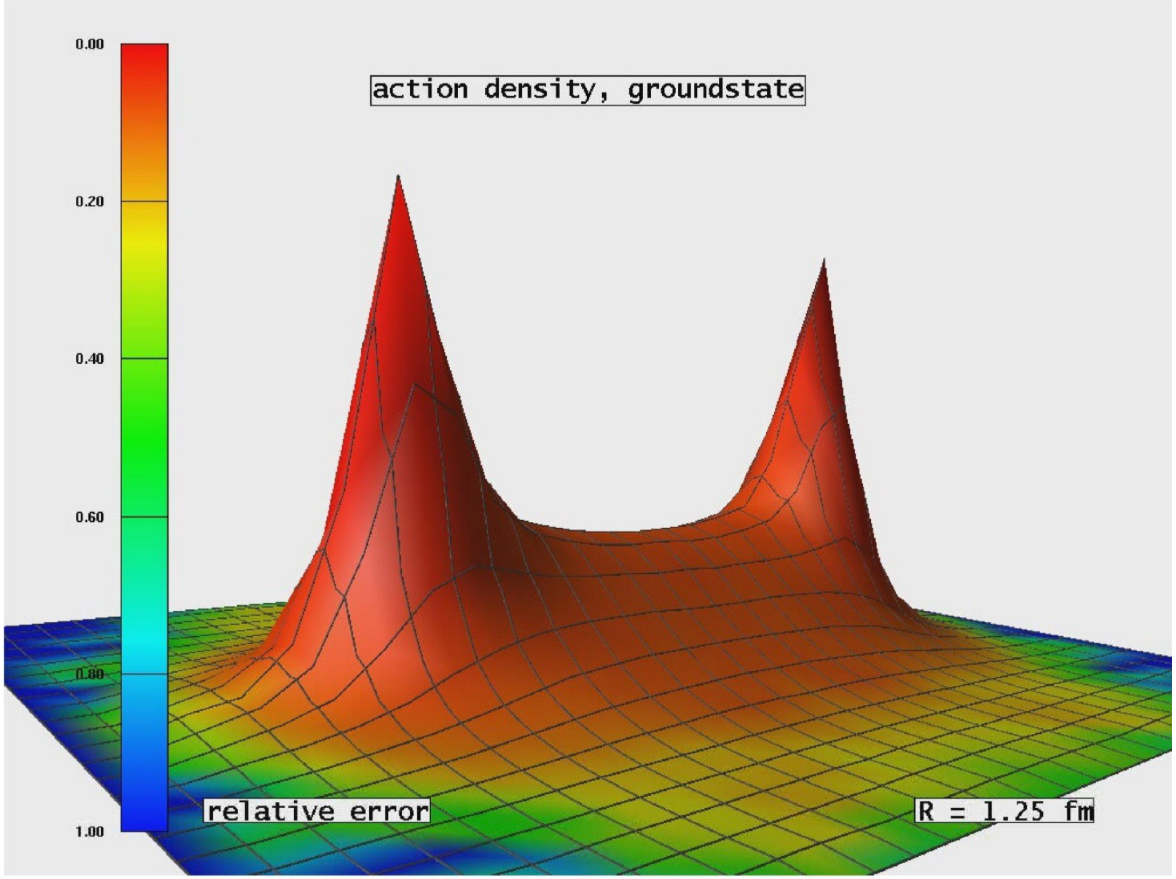
Light quarks & Confinement

- Folklore … *Hall-D Conceptual Design Report(5)*

“The color field lines between a quark and an anti-quark form flux tubes.

A unit area placed midway between the quarks and perpendicular to the line connecting them intercepts a constant number of field lines, independent of the distance between the quarks.

This leads to a constant force between the quarks – and a large force at that, equal to about 16 metric tons.”



Light quarks & Confinement

- Problem:
  16 tonnes of force
  makes a lot of pions.

Light quarks & Confinement

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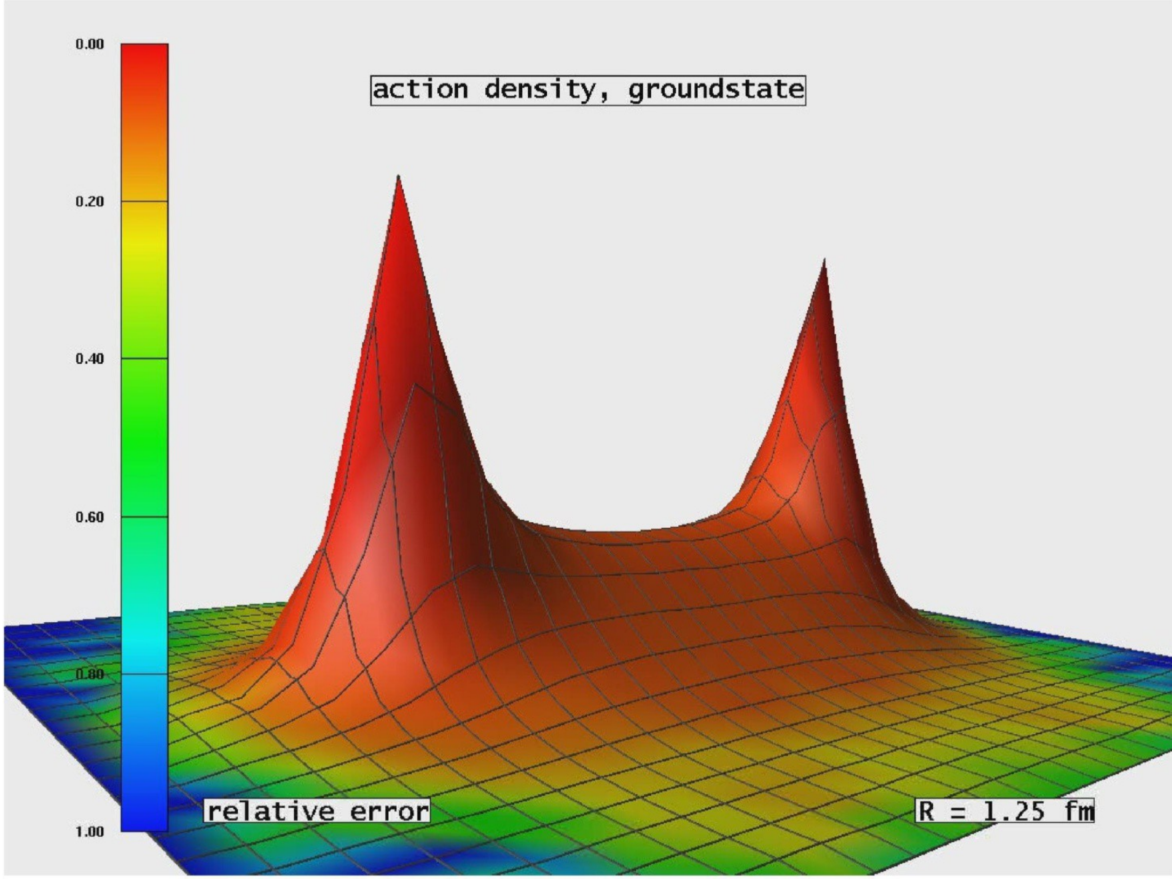
. Bali et al., *PoS LAT2005 (2006) 308*

Light quarks & Confinement

- In the presence of light quarks, *pair creation seems to occur non-localized and instantaneously*
- No flux tube in a theory with light-quarks.
- ***Flux-tube is not the correct paradigm for confinement in hadron physics***



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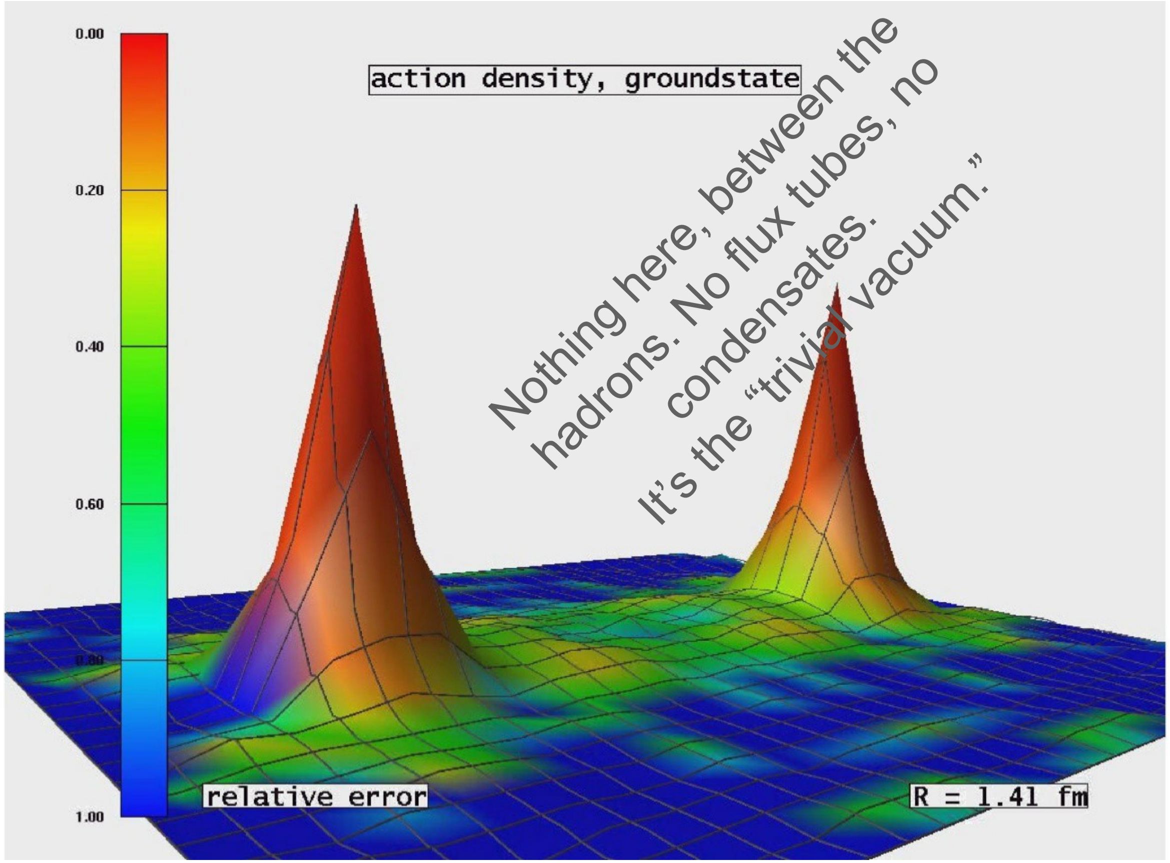
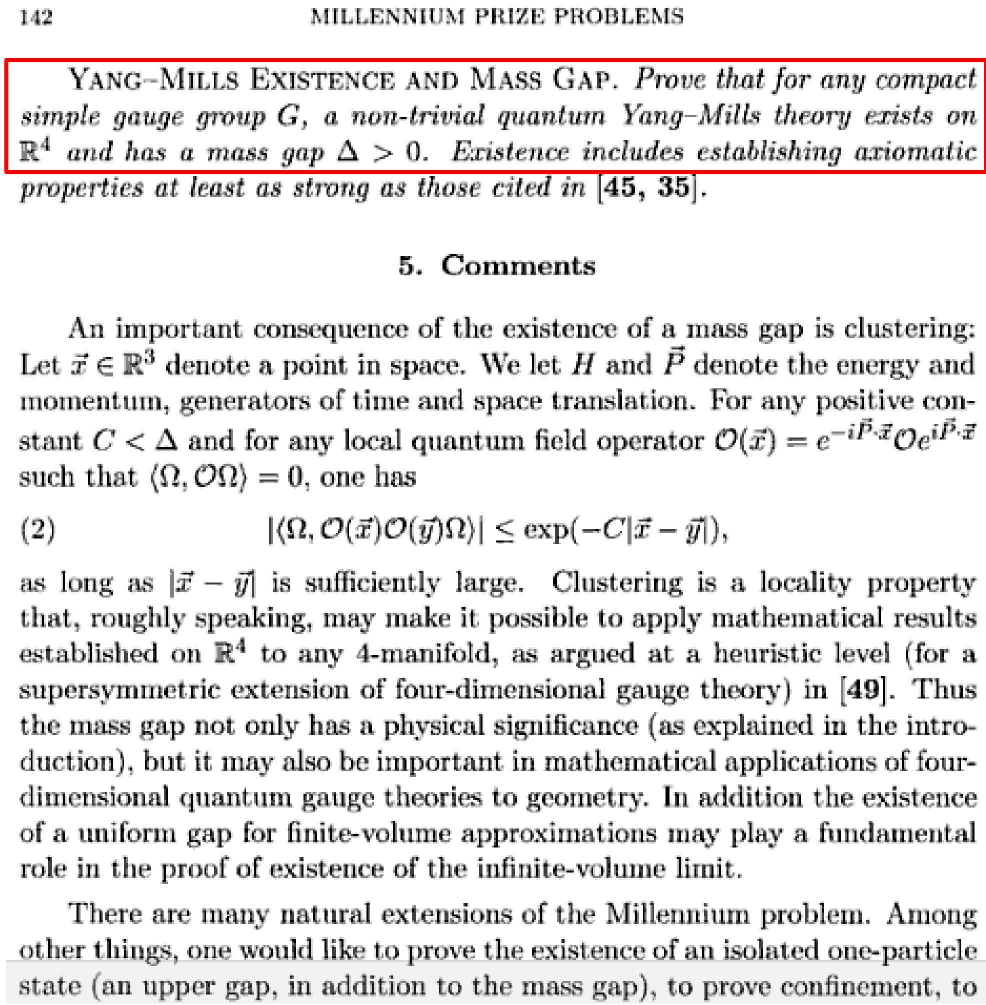


Nothing here, between the hadrons. No flux tubes, no condensates. It's the "trivial vacuum."

- *Existence of mass-gap in pure-gauge theory*
- Strong evidence supporting this conjecture: lQCD predicts *Δ* ~ 1.5 GeV
- However, with

  $\Delta 2/m\pi 2 > 100$,

  can mass-gap in pure Yang-Mills play any role in understanding confinement when dynamical chiral symmetry breaking (DCSB) ensures existence of an almost-massless strongly-interacting excitation in our Universe?
- If *answer is* not *simply* ***no***, then it must be that one cannot claim to provide an understanding of confinement without simultaneously explaining its connection with DCSB.
- *Pion must play critical role in any explanation of confinement; and any discussion that omits reference to the pion's role is practically irrelevant.*



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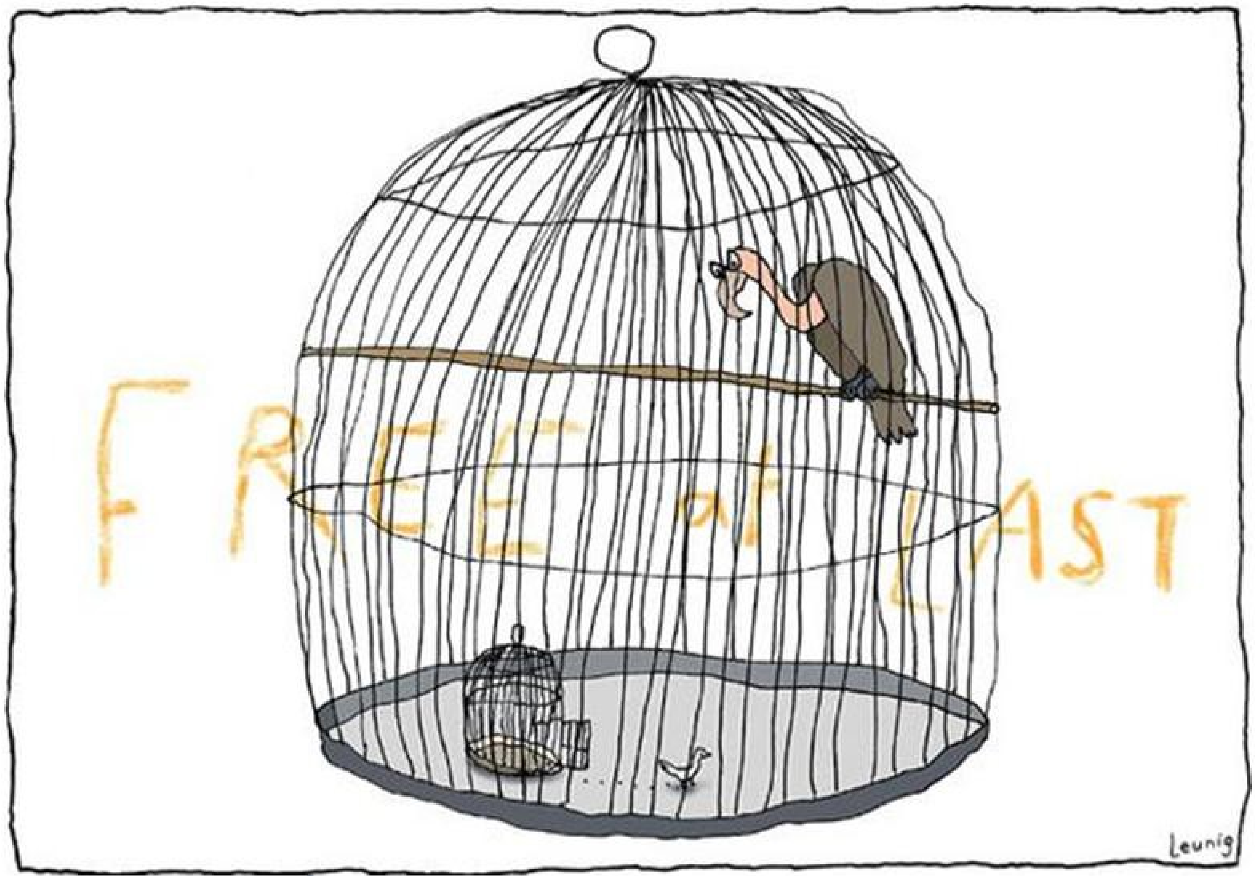
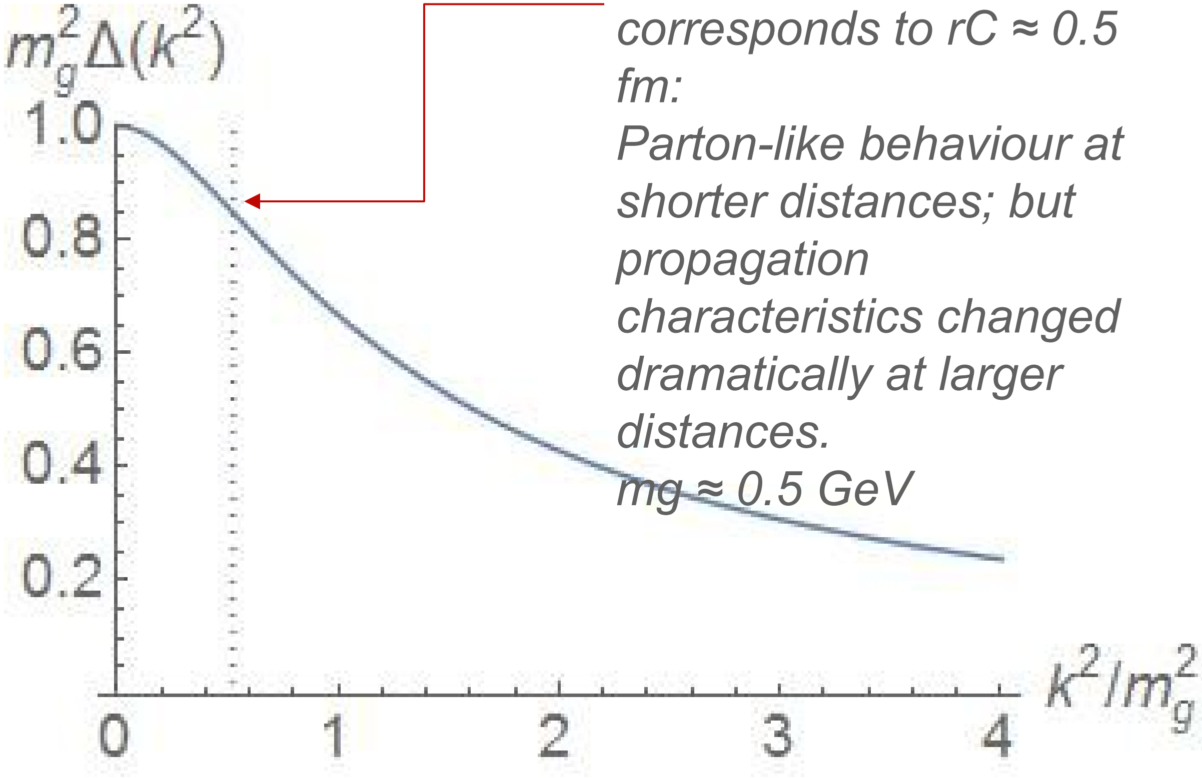


Confinement is dynamical

Confinement

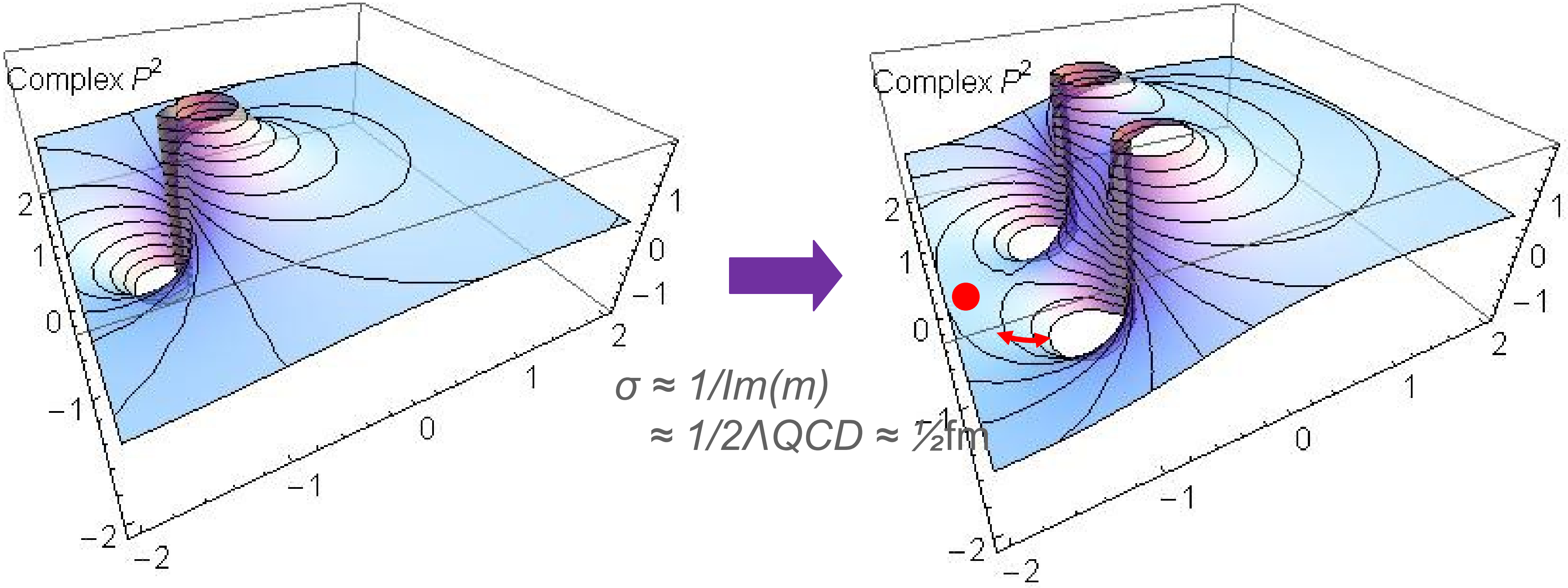
- All continuum and lattice solutions for Landau-gauge gluon & quark propagators exhibit an inflection point in *k2*
- Violate reflection positivity = sufficient for confinement
- Such states have negative norm
- All observable states of a physical Hamiltonian have positive norm
- Negative norm states are not observable



Inflexion point corresponds to rC ≈ 0.5 fm:
Parton-like behaviour at shorter distances; but propagation characteristics changed dramatically at larger distances.
mg ≈ 0.5 GeV

Confinement

- Meaning …



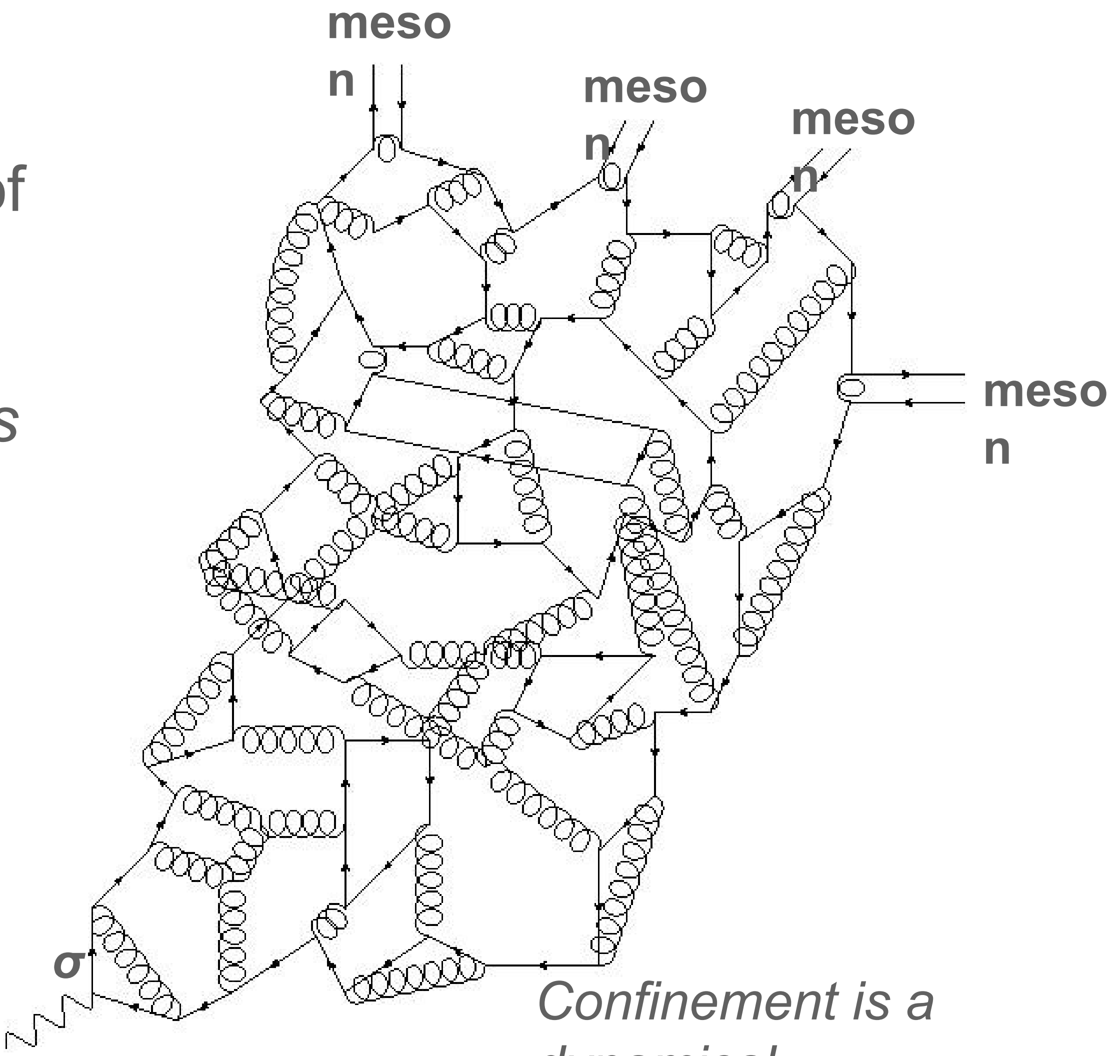
σ ≈ 1/Im(m)
≈ 1/2ΛQCD ≈ ½fm

Real-particle mass-pole splits, moving into pair(s) of complex conjugate singularities, (or qualitatively analogous structures chracterised by a dynamically generated mass-scale)

Propagation described by rapidly damped wave & hence state cannot exist in observable spectrum

Quark Fragmentation

- A quark begins to propagate
- But after each “step” of length ***σ***, on average, an interaction occurs, so that the quark *loses* its identity, sharing it with other partons
- Finally, a cloud of partons is produced, which coalesces into colour-singlet final states



Confinement is a dynamical phenomenon!

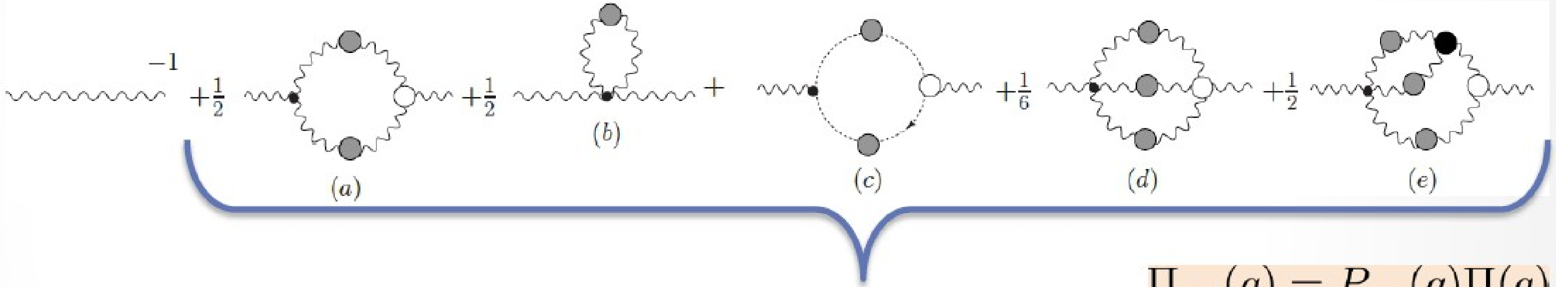
Confinement in Thessaloniki

Outcome of discussions at Confinement XII

  - Agreed position of Bali, Brambilla, Petreczky, Roberts:

- The flux tube measured in numerical simulations of lQCD with static quarks has zero relevance to confinement in the purely light-quark realm of QCD.
- There is zero knowledge of the strength or extension of a flux tube between a static-quark and any light-quark. Indeed, it is impossible to define such a flux tube. It is impossible to compute or even define a flux-tube between a light-quark source and light-quark sink.
- Since the vast bulk of visible matter is constituted from light valence quarks, with no involvement of even an accessible heavy quark, then the flux tube picture is not the correct paradigm for confinement in hadron physics.
- Confinement in hadron physics is a dynamical phenomenon, intimately connected with the fragmentation effect. It cannot be comprehended without simultaneously understanding dynamical chiral symmetry breaking, which is the origin of a near-zero mass hadron (pion).

$$\Delta^{-1}_{\mu\nu}(q) =$$



$$\Pi_{\mu\nu}(q)$$

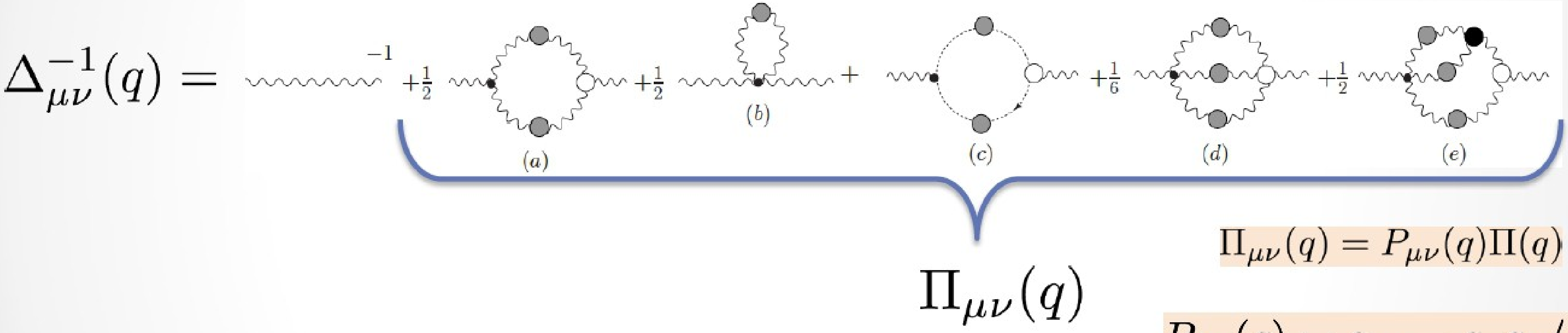
$$\Pi_{\mu\nu}(q) = P_{\mu\nu}(q)\Pi(q)$$

$$P_{\mu\nu}(q) = g_{\mu\nu} - q_\mu q_\nu/q^2$$

Gluon Gap Equation

Non-perturbative comparison of QCD effective charges, A.C. Aguilar, D. Binosi, J. Papavassiliou and
J. Rodriguez-Quintero, Phys. Rev. D80 (2009) 085018

Gluon Gap Equation



$$\Pi_{\mu\nu}(q) = P_{\mu\nu}(q)\Pi(q)$$

$$P_{\mu\nu}(q) = g_{\mu\nu} - q_\mu q_\nu/q^2$$

- Pinch-technique + background field method … reordering of diagrammatic summations in the self-energy – *Πμν* – ensures that subclusters are individually transverse and gluon-loop and ghost-loop contributions are separately transverse
- STIs → WGTIs
- Enables systematic analysis and evaluation of truncations and straightforward comparison of results with those of lQCD

Bridging a gap between continuum-QCD and ab initio predictions of hadron observables, D. Binosi *et al.*, arXiv:1412.4782 [nucl-th], Phys. Lett. B742 (2015) 183-188

In QCD: Gluons also become massive!

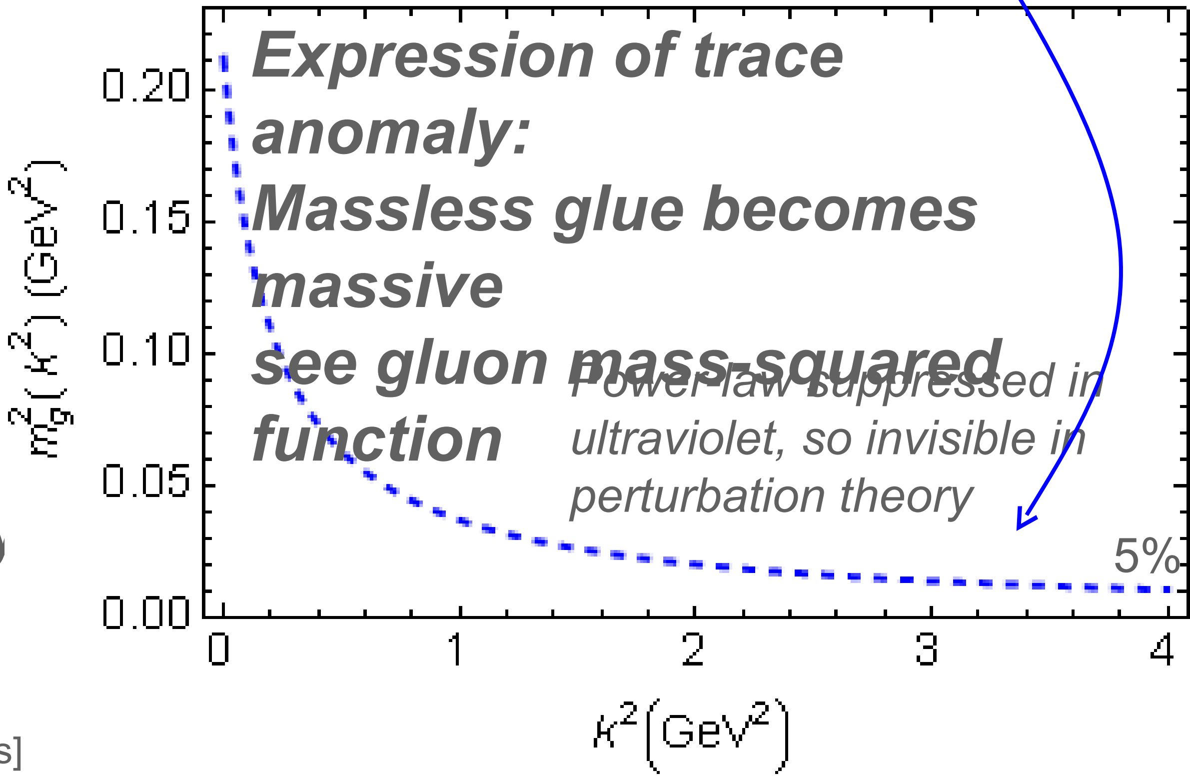
- Running gluon mass

$$d(k^2) = \frac{\alpha(\zeta)}{k^2 + m_g^2(k^2;\zeta)}$$

$$\alpha_s(0) = 2.77 \approx 0.9\pi\,,\ m_g^2(0) = (0.46\,\text{GeV})^2$$

- Gluons are ***cannibals*** – a particle species whose members become massive by eating each other!

$$m_g^2(k^2) = \frac{\mu_g^4}{\mu_g^2 + k^2}$$



Expression of trace anomaly:

Massless glue becomes massive

see gluon mass-squared function

Power-law suppressed in ultraviolet, so invisible in perturbation theory

Interaction model for the gap equation, S.-x.Qin *et al.*, arXiv:1108.0603 [nucl-th], Phys. Rev. C **84** (2011) 042202(R) [5 pages]

Massive Gauge Bosons!

- Gauge boson cannibalism

  … a new physics frontier … within the Standard Model
- Asymptotic freedom means

  … ultraviolet behaviour of QCD is controllable
- Dynamically generated masses for gluons and quarks means that QCD dynamically generates its own infrared cutoffs
  - Gluons and quarks with

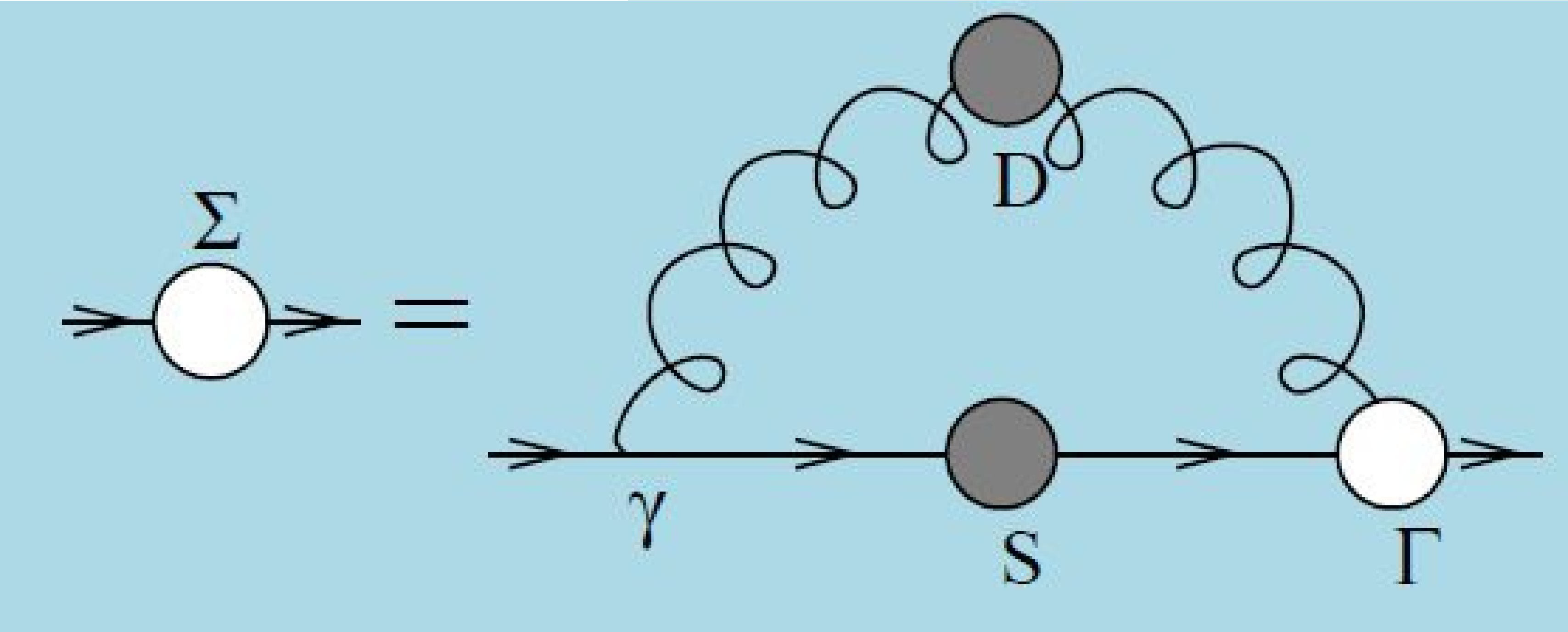
    wavelength $\lambda > 2/\text{mass} \approx 1$ fm

  decouple from the dynamics … ***Confinement?!***
- How does that affect observables?
  - It will have an impact in any continuum study
  - Probably plays a role in gluon saturation …

  In fact, a harbinger of gluon saturation?

Electron Ion Collider: The Next QCD Frontier

$$S(p) = \frac{Z(p^2)}{i\gamma\cdot p + M(p^2)}$$

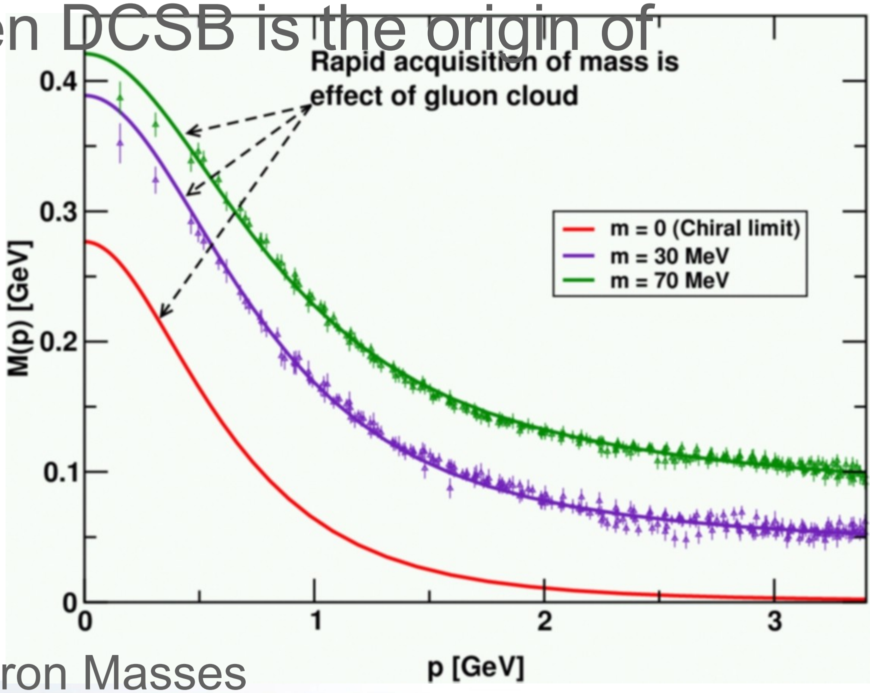


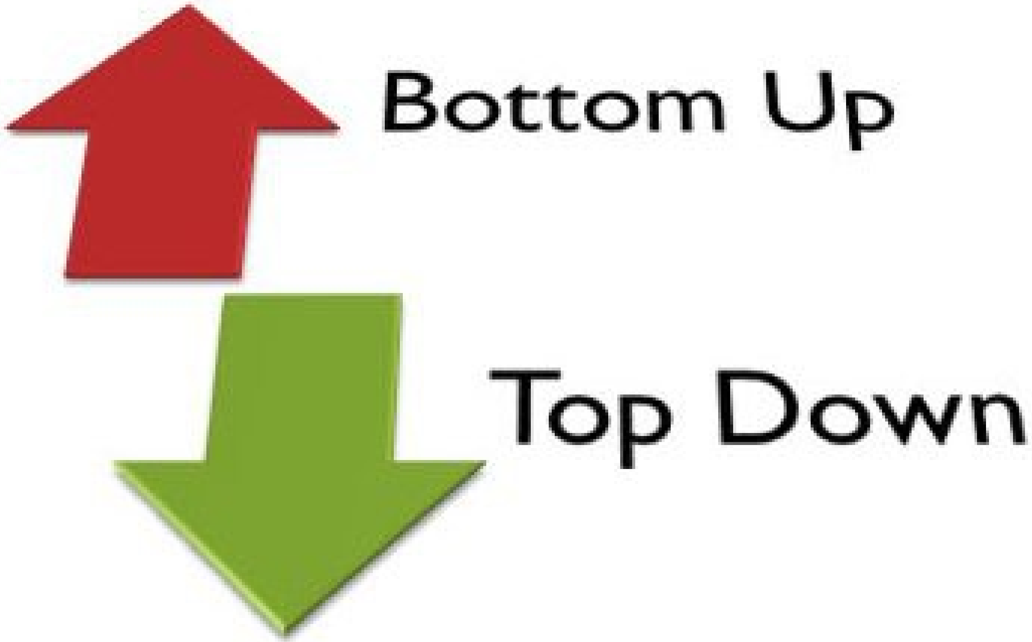
Quark Gap Equation

DCSB

- Dynamical chiral symmetry breaking (DCSB) is a critical emergent phenomenon in QCD
- Expressed in hadron wave functions not in vacuum condensates
- Contemporary theory indicates that DCSB is responsible for more than 98% of the visible mass in the Universe; namely, given that classical massless-QCD is a conformally invariant theory, then DCSB is the origin of *mass from nothing*.
- ***Dynamical***, not spontaneous
  - Add nothing to *QCD* , *No Higgs field, nothing!*

    Effect achieved purely through quark+gluon dynamics.





Continuum-QCD & *ab initio* predictions

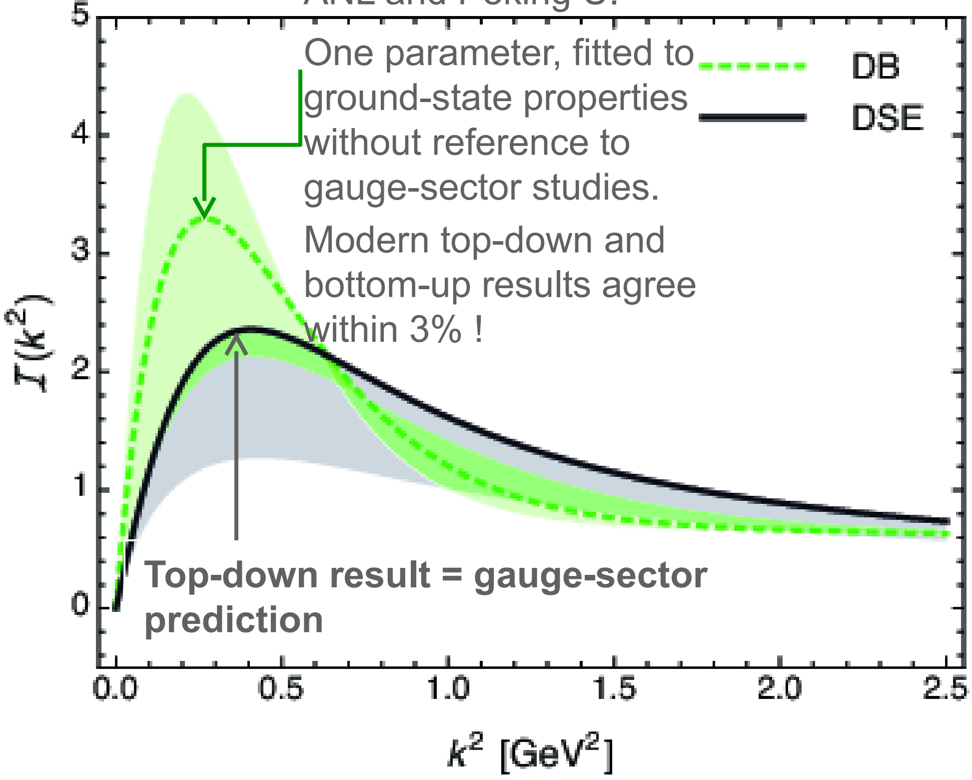
Top down & Bottom up

Bridging a gap between continuum-QCD & ab initio predictions of hadron observables

D. Binosi (Italy), L. Chang (Australia), J. Papavassiliou (Spain),
C. D. Roberts (US), arXiv:1412.4782 [nucl-th] , *Phys. Lett. B* ***742*** *(2015) 183*

- *<u>Top-down approach</u>* – ab initio *computation of the interaction via direct analysis of the gauge-sector gap equations*
- *<u>Bottom-up scheme</u> – infer interaction by fitting data within a well-defined truncation of the matter sector DSEs that are relevant to bound-state properties.*
- *Serendipitous collaboration, conceived at one-week ECT* Workshop on* DSEs in Mathematics and Physics, *has united these two approaches*

Modern kernels and interaction, developed at ANL and Peking U.



One parameter, fitted to ground-state properties without reference to gauge-sector studies.

Modern top-down and bottom-up results agree within 3% !

Top-down result = gauge-sector prediction

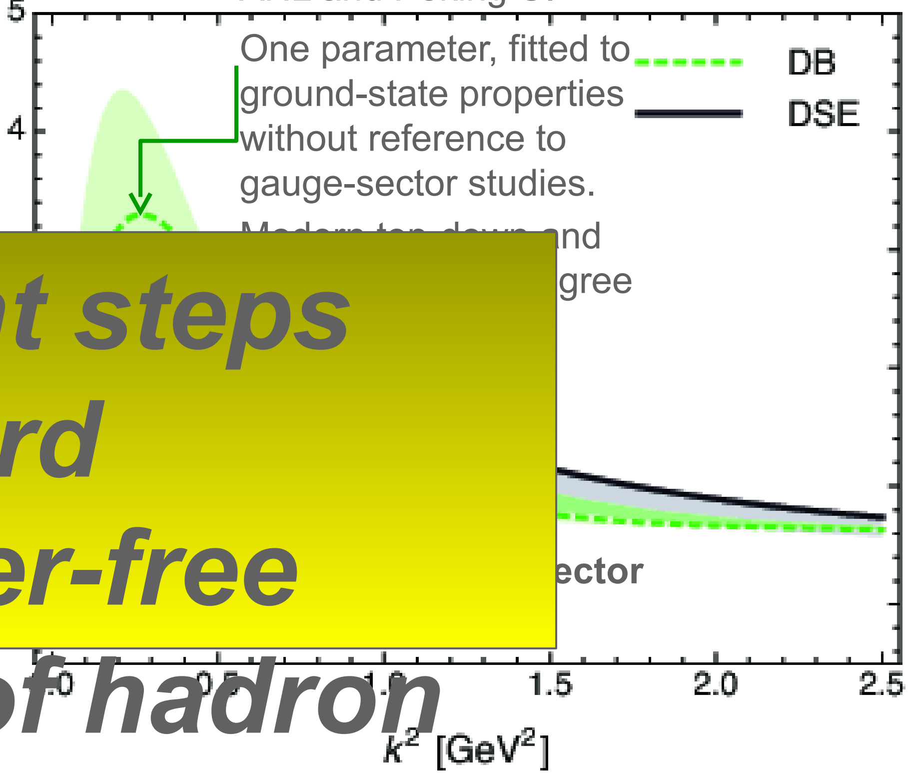
– *Interaction predicted by modern analyses of QCD's gauge sector coincides with that required to describe ground-state observables using the sophisticated matter-sector ANL-PKU DSE truncation*

Bridging a gap between continuum-QCD & ab initio predictions of hadron observables

Top down & Bottom up

D. Binosi (Italy), L. Chang (Australia), J. Papavassiliou (Spain),
C. D. Roberts (US), arXiv:1412.4782 [nucl-th] , *Phys. Lett. B* ***742*** *(2015) 183*

- *Bottom-up scheme – infer interaction by fitting data within a well-defined truncation of the matter sector DSEs that are relevant to bound-state properties.*
- *Top-down approach* – ab initio computation *of the interaction via direct analysis of the gauge-sect*
- *Serendipito*
  one-week E
  Mathematic
  two approa



Modern kernels and interaction, developed at ANL and Peking U.

One parameter, fitted to ground-state properties without reference to gauge-sector studies.

Modern top-down and
gree

ector

Significant steps toward parameter-free prediction of hadron properties

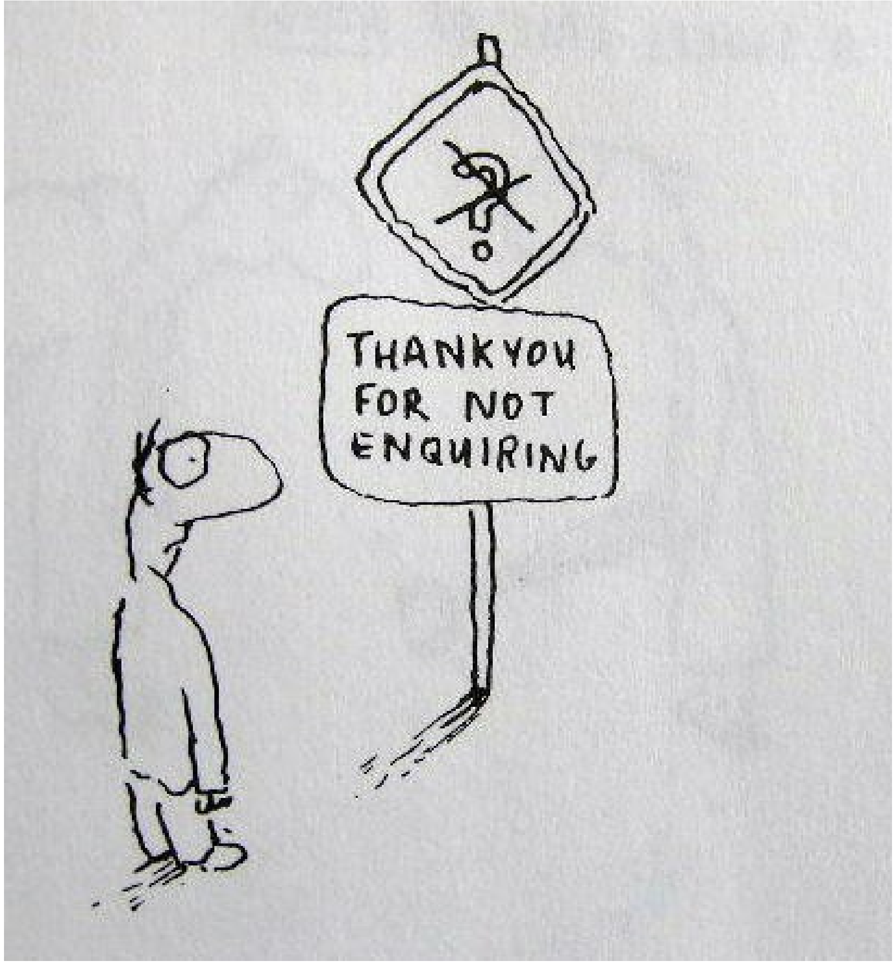
– Interaction predicted by modern analyses of QCD's gauge sector coincides with that required to describe ground-state observables using the sophisticated matter-sector ANL-PKU DSE truncation

Natural constraints on the gluon-quark vertex
Binosi, Chang, Papavassiliou, Qin, Roberts,
arXiv:1609.02568 [nucl-th]

Reconciliation demands dressed-gluon-quark vertex

- Significant progress since 2009:
  dressed *Γμ* in gap- and Bethe-Salpeter equations …
- In principle, ∃ unique form of *Γμ* , but it's still obscure.
- To improve this situation, used the top-down/bottom-up RGI running-interaction
  - Computed gap equation solutions with
    1,660,000 distinct *Ansätze* for *Γμ*
- Each one of the solutions tested for compatibility with three physical criteria
- Remarkably, merely 0.55% of solutions survive the test

þ Even a small selection of observables places extremely tight bounds on the domain of acceptable, realistic vertex *Ansätze*



Enigma of Mass

Maris, Roberts and Tandy
*nucl-th/9707003, Phys.Lett. B**420** (1998) 267-273*

Pion's Goldberger -Treiman relation

- Pion's Bethe-Salpeter amplitude

  Solution of the Bethe-Salpeter equation

$$\begin{aligned}\Gamma_{\pi^j}(k;P) \quad &= \quad \tau^{\pi^j}\gamma_5\Big[iE_\pi(k;P)+\gamma\cdot P F_\pi(k;P)\\ &\quad +\gamma\cdot k\,k\cdot P\,G_\pi(k;P)+\sigma_{\mu\nu}\,k_\mu P_\nu\,H_\pi(k;P)\Big]\end{aligned}$$

- Dressed-quark propagato $S(p)=\dfrac{1}{i\gamma\cdot p\,A(p^2)+B(p^2)}$

- Axial-vector Ward-Takahashi identity entails

$$f_\pi E_\pi(k;P=0) \quad = \quad B(k^2)$$

Owing to DCSB & Exact in Chiral QCD

***Miracle*: two body problem solved, almost completely, once solution of one body problem is known**

Rudimentary version of this relation is apparent in Nambu's Nobel Prize work

Model independent
Gauge independent
Scheme independent

$$f\pi\, E\pi(p2) = B(p2)$$

The most fundamental expression of Goldstone's Theorem and DCSB

Rudimentary version of this relation is apparent in Nambu's Nobel Prize work

Model independent
Gauge independent
Scheme independent

$$f\pi\ E\pi(p2) \Leftrightarrow B(p2)$$

Pion exists if, and only if, mass is dynamically generated

Rudimentary version of this relation is apparent in Nambu's Nobel Prize work

Model independent
Gauge independent
Scheme independent

$$f\pi \; E\pi(p2) \Leftrightarrow B(p2)$$

This is why *mπ=0*
in the absence of a Higgs mechanism

Enigma of mass

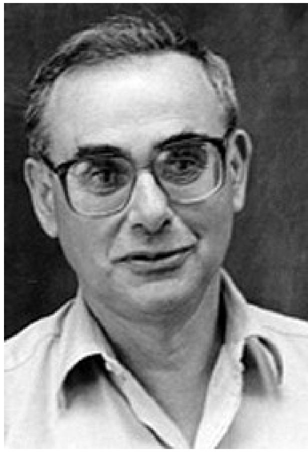
$$f_\pi E_\pi(p2) = B(p2)$$

This algebraic identity is why QCD's pion is massless in the chiral limit

- The quark level Goldberger-Treiman relation shows that DCSB has a very deep and far reaching impact on physics within the strong interaction sector of the Standard Model; viz.,

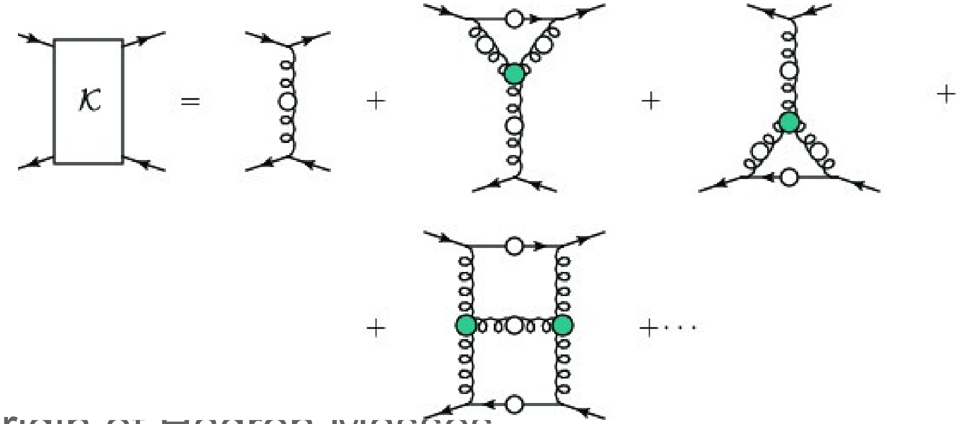
Goldstone's theorem is fundamentally an expression of equivalence between the one-body problem and the two-body problem in the pseudoscalar channel.

- This emphasises that Goldstone's theorem has a pointwise expression in QCD
- Hence, pion properties are an almost direct measure of the dressed-quark mass function.
- Thus, enigmatically, the properties of the *massless* pion are the cleanest expression of the mechanism that is responsible for almost all the visible mass in the universe.



Pion masslessness

- Renormalisation scale: $\zeta = 2\text{GeV} =: \zeta_2$
- Pion's Poincaré-invariant mass and Poincaré-covariant wave function are obtained by solving a Bethe-Salpeter equation.
- This is a scattering problem
- In chiral limit
  - two massless fermions interact via exchange of massless gluons … initial system is massless; and it remains massless at every order in perturbation theory
- But, complete the calculation using an enumerable infinity of dressings and scatterings



Munczek, H. J., Phys. Rev. D 52 (1995) pp. 4736-4740
Bender, A., Roberts, C.D. and von Smekal, L., Phys. Lett. B 380 (1996) pp. 7-12
Maris, P. , Roberts, C.D. and Tandy, P.C., Phys. Lett. B 420 (1998) pp. 267-273

Pion masslessness

- Produces a coupled set of gap- and Bethe-Salpeter equations
  - Bethe-Salpeter Kernel:
    - valence-quarks with a momentum-dependent running mass produced by self-interacting gluons, which have given themselves a running mass
    - Interactions of arbitrary but enumerable complexity involving these “basis vectors”
  - Chiral limit:
    - Algebraic proof that, at any finite order in a symmetry-preserving construction of the kernels for the gap (quark dressing) and Bethe-Salpeter (bound-state) equations, there is a precise cancellation between the mass-generating effect of dressing the valence-quarks and the attraction introduced by the scattering events
    - Cancellation guarantees that the simple system, which began massless, becomes a complex system, with a nontrivial bound-state wave function attached to a pole in the scattering matrix, which remains at P2=0 … remains massless
- Quantum field theory statement: in the pseudsocalar channel, the dynamically generated mass of the two fermions is precisely cancelled by the attractive interactions between them – iff –

$$f_\pi\, E_\pi(p2) =$$

Pion masslessness

DCSB Paradigm

$$\langle\pi(q)|\theta_0|\pi(q)\rangle \overset{\zeta\gg\zeta_2}{=} \langle\pi(q)|\tfrac{1}{4}\beta(\alpha(\zeta))G^a_{\mu\nu}G^a_{\mu\nu}|\pi(q)\rangle$$

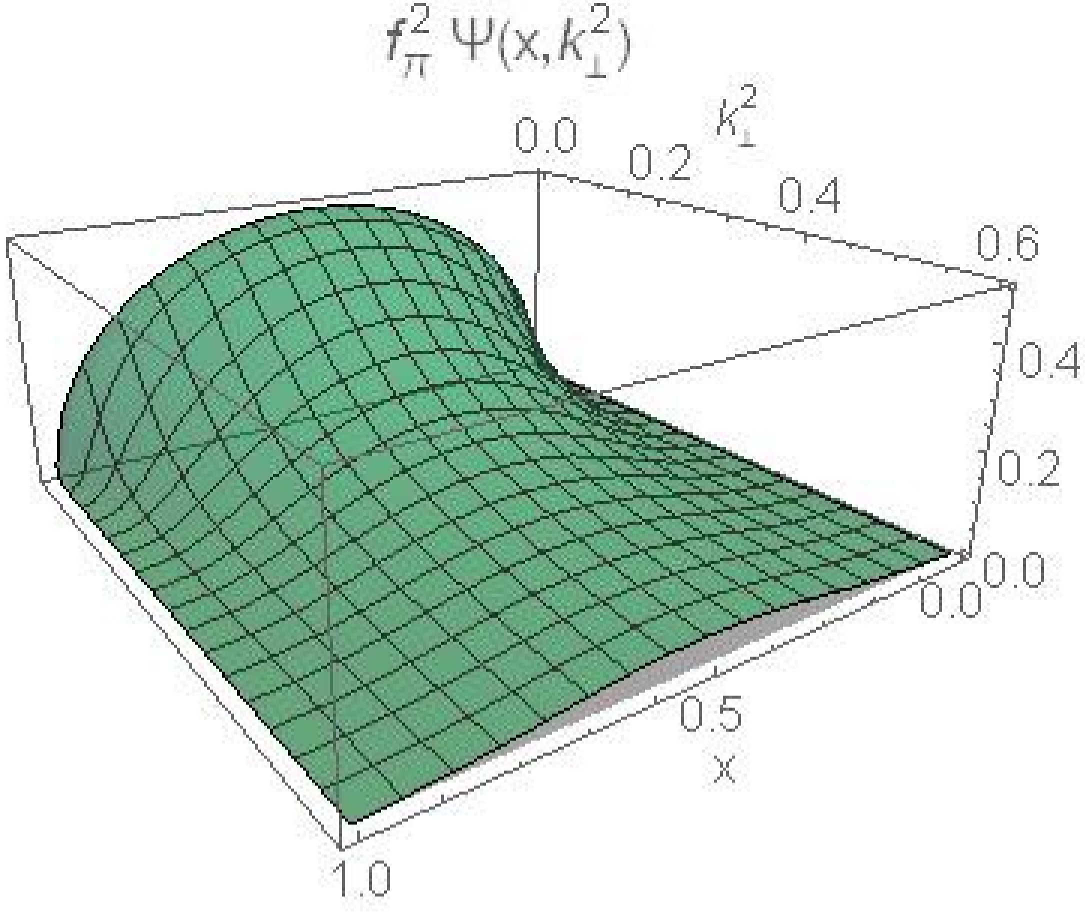
$$\overset{\zeta\simeq\zeta_2}{\to} \langle\pi(q)| \sum_{f=u,d} M_f(\zeta)\,\bar{\mathcal{Q}}_f(\zeta)\mathcal{Q}_f(\zeta) + \tfrac{1}{4}[\beta(\alpha(\zeta))\mathcal{G}^a_{\mu\nu}\mathcal{G}^a_{\mu\nu}]_{2\mathrm{PI}}|\pi(q)\rangle$$

- Parton-basis chiral-limit expression of the expectation-value of the trace-anomaly in the pion at $\zeta \gg \zeta 2$
- Metamorphoses into a new expression, written in terms of a nonperturbatively-dressed quasi-particle basis
  - 1st term = *positive* = one-body dressing content of the trace anomaly … Plainly, a massless valence-quark acquiring a large mass through interactions with its own gluon field is an expression of the trace-anomaly in the one-body subsector of the complete pion wave function
  - 2nd term = *negative* (attraction) = 2-particle-irreducible scattering event content of the scale-anomaly … Plainly, acquires a scale because the couplings, and the gluon- and quark-propagators in the 2PI processes have all acquired a mass-scale
- Away from the chiral limit, and in other channels, the cancellation is incomplete.

Observing *Mass*

Observing Mass

- Goldberger-Treiman relations entail that on $m \simeq 0$, dressed-quark mass function (almost) completely determines χ_π (wave function)
- χ_π can be projected onto the light-front
  - Object thus obtained is strictly a probability amplitude and moments of a probability measure are truly observable.
  - Consequently, there is a mathematically strict sense in which moments of the dressed-quark mass function are observable.
  - Additionally, *e.g*. generalised parton distributions can rigorously be defined as an overlap of light-front wave functions
- Practically, the mass function can be "measured" because it influences and determines a vast array of experimental observables
- *In this sense, M(p2), microscopic expression of trace anomaly, is observable at modern facilities, in measurements of*
  - *form factors (elastic and transition),*
  - *PDAs, PDFs, GPDs, TMDs, …*
- *Examples …*



Pion's Wave Function

Imaging dynamical chiral symmetry breaking: pion wave function on the light front, Lei Chang, et al., arXiv:1301.0324 [nucl-th], Phys. Rev. Lett. **110** (2013) 132001 (2013) [5 pages].

Pion’s valence-quark Distribution Amplitude

- 2012 … methods were developed that enable direct computation of the pion’s light-front wave function
- *φπ(x)* = twist-two parton distribution amplitude = projection of the pion’s Poincaré-covariant wave-function onto the light-front

$$\varphi_\pi(x) = Z_2 \mathrm{tr}_{CD} \int \frac{d^4k}{(2\pi)^4} \delta(n \cdot k - x n \cdot P)\, \gamma_5 \gamma \cdot n\, S(k) \Gamma_\pi(k; P) S(k - P)$$

currently available, which unifies matter & gauge sectors

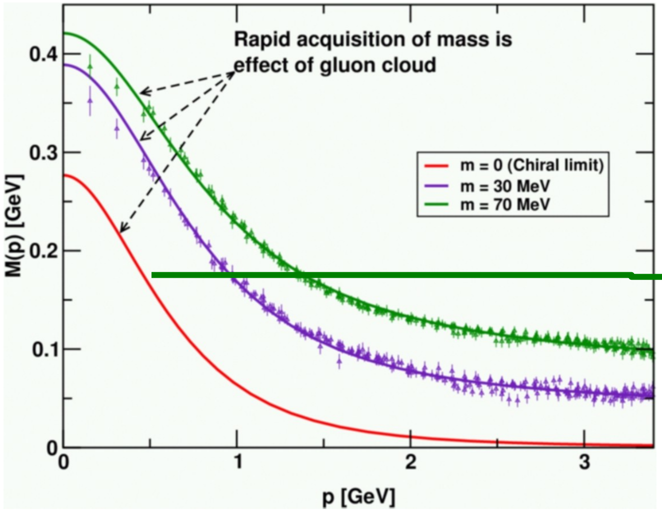
ϕπ(x) ∝ *xα (1-x)α*, with *α≈0.5*

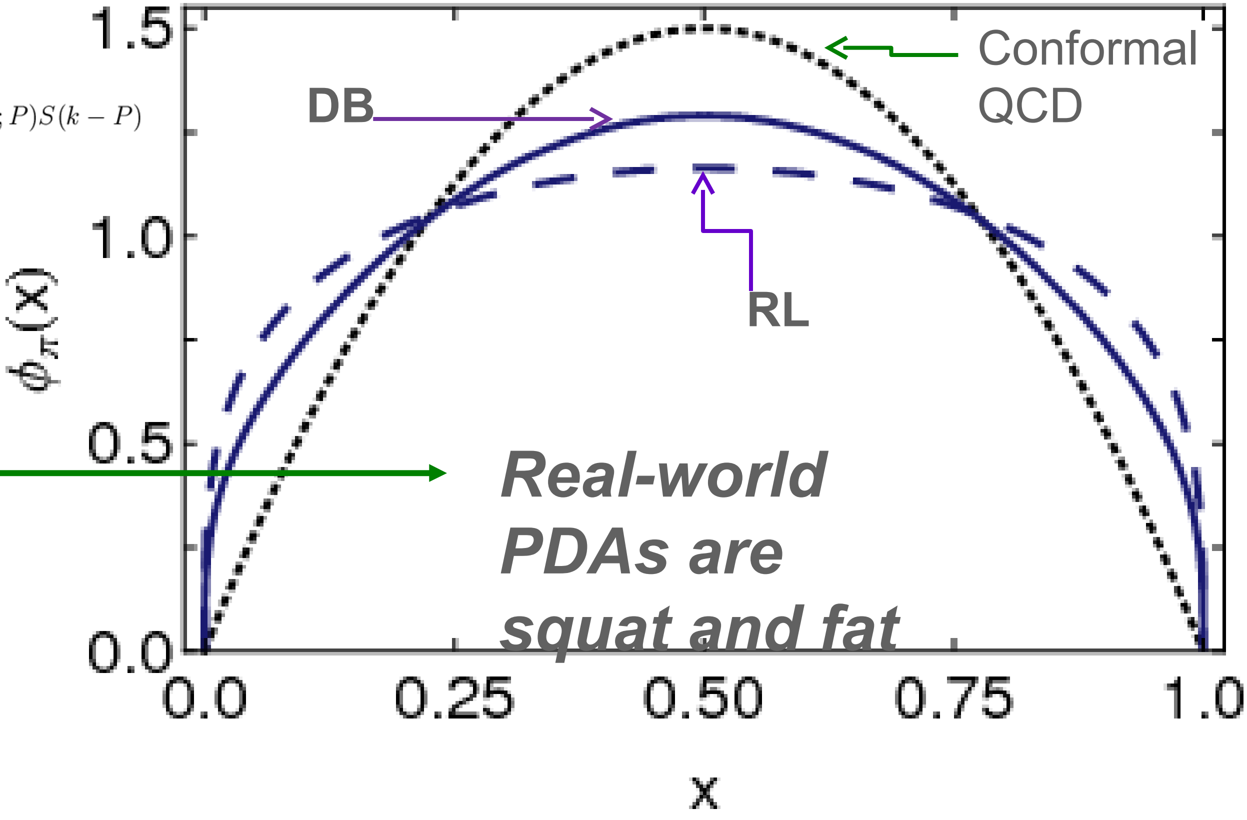
Imaging dynamical chiral symmetry breaking: pion wave function on the light front, Lei Chang, *et al*., arXiv:1301.0324 [nucl-th], Phys. Rev. Lett. **110** (2013) 132001 (2013) [5 pages].

Pion's valence-quark Distribution Amplitude

- Continuum-QCD prediction:
  marked broadening of *φπ*(x), which owes to DCSB

$$\varphi_\pi(x) = Z_2 \mathrm{tr}_{CD} \int \frac{d^4k}{(2\pi)^4} \delta(n\cdot k - x n\cdot P)\, \gamma_5 \gamma\cdot n\, S(k) \Gamma_\pi(k;P) S(k-P)$$





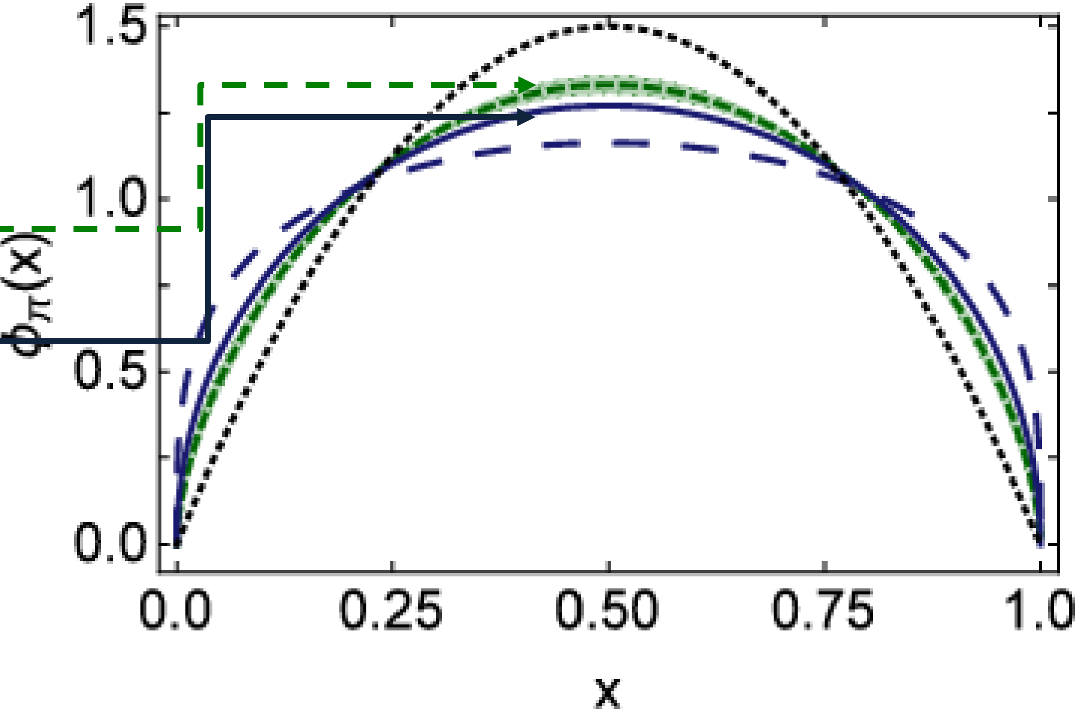
Real-world PDAs are squat and fat

Pion distribution amplitude from lattice-QCD I. C. Cloët *et al., arXiv:1306.2645 [nucl-th], Phys. Rev. Lett. 111 (2013) 092001 [5 pages]*

Flavour symmetry breaking in the kaon parton distribution amplitude, Chao Shi *et al., arXiv:1406:3353 [nucl-th], Phys. Lett. B 738 (2014) pp. 512–518*

Lattice-QCD & Pion's valence-quark PDA

- Isolated dotted curve = conformal QCD
- Green curve & band = result inferred from the single pion moment computed in lattice-QCD
- Blue solid curve = DSE prediction obtained with DB kernel
- DSE & lQCD predictions are practically indistinguishable

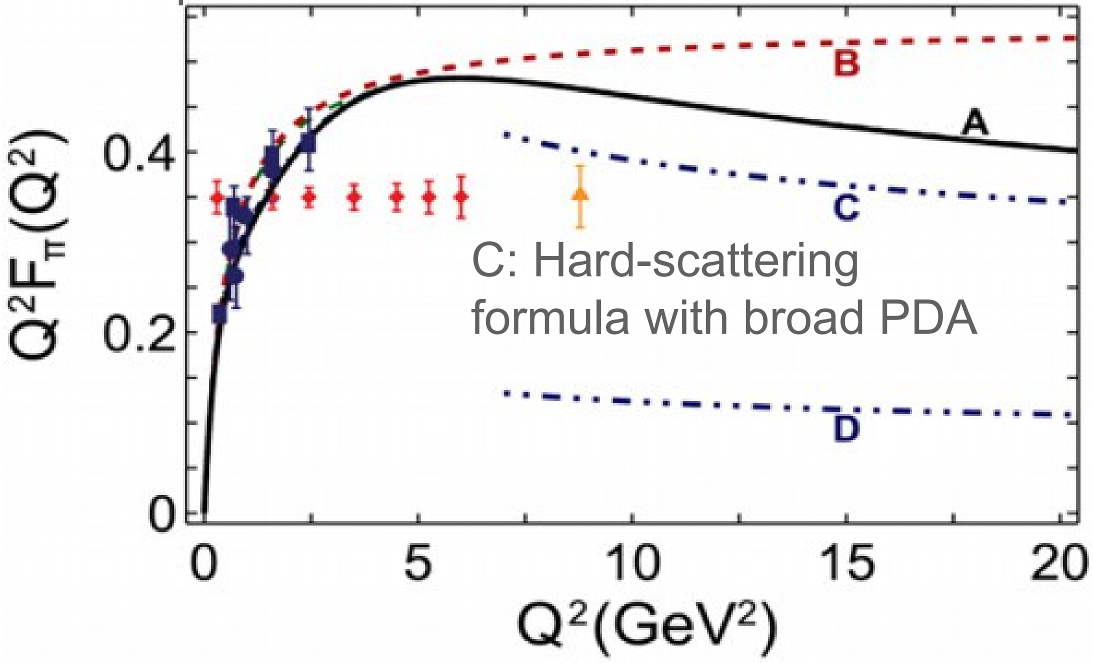


Pion electromagnetic form factor at spacelike momenta
L. Chang *et al., arXiv:1307.0026 [nucl-th],*
Phys. Rev. Lett. ***111****, 141802 (2013)*

Pion's electromagnetic form factor

- Broadening has enormous impact on understanding *Fπ(Q2)*

A: Internally-consistent DSE prediction



C: Hard-scattering formula with broad PDA

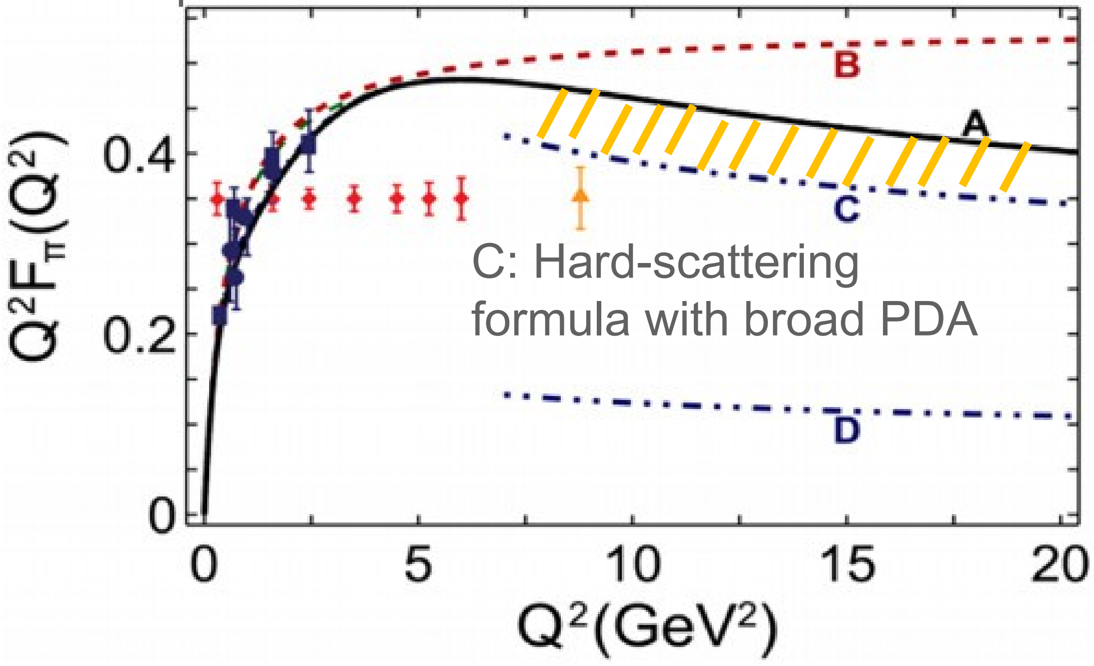
Figure 2.2: *Existing (dark blue) data and projected (red, orange) uncertainties for future data on the pion form factor. The solid curve (A) is the QCD-theory prediction bridging large and short distance scales. Curve B is set by the known long-distance scale—the pion radius. Curves C and D illustrate calculations based on a short-distance quark-gluon view.*

Pion electromagnetic form factor at spacelike momenta
L. Chang, I. C. Cloët, C. D. Roberts, S. M. Schmidt and P. C. Tandy, arXiv:1307.0026 [nucl-th], Phys. Rev. Lett. 111, 141802 (2013)

Pion's electromagnetic form factor

- Broadening has enormous impact on understanding *Fπ(Q2)*
- Appears that JLab12 is within reach of first verification of a QCD hard-scattering formula

A: Internally-consistent DSE prediction



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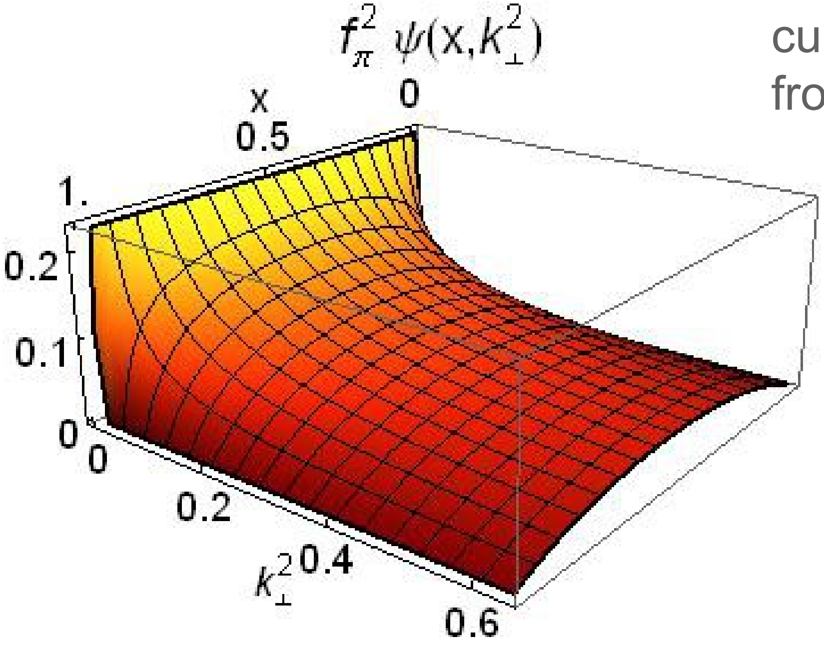
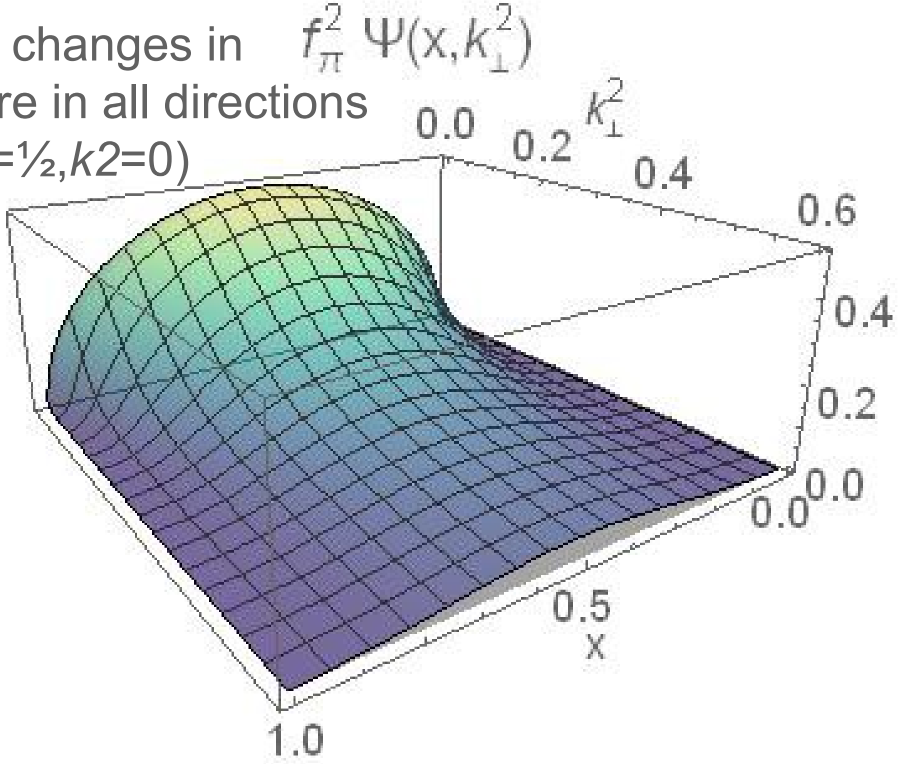
Light-front wave function

- PDAs computed; but we're still refining continuum methods in order to compute the complete leading-twist wave function
- Realistic wave function at *ζ2* will look like this …



Conformal limit

Marked changes in curvature in all directions from (*x*=½,*k2*=0)

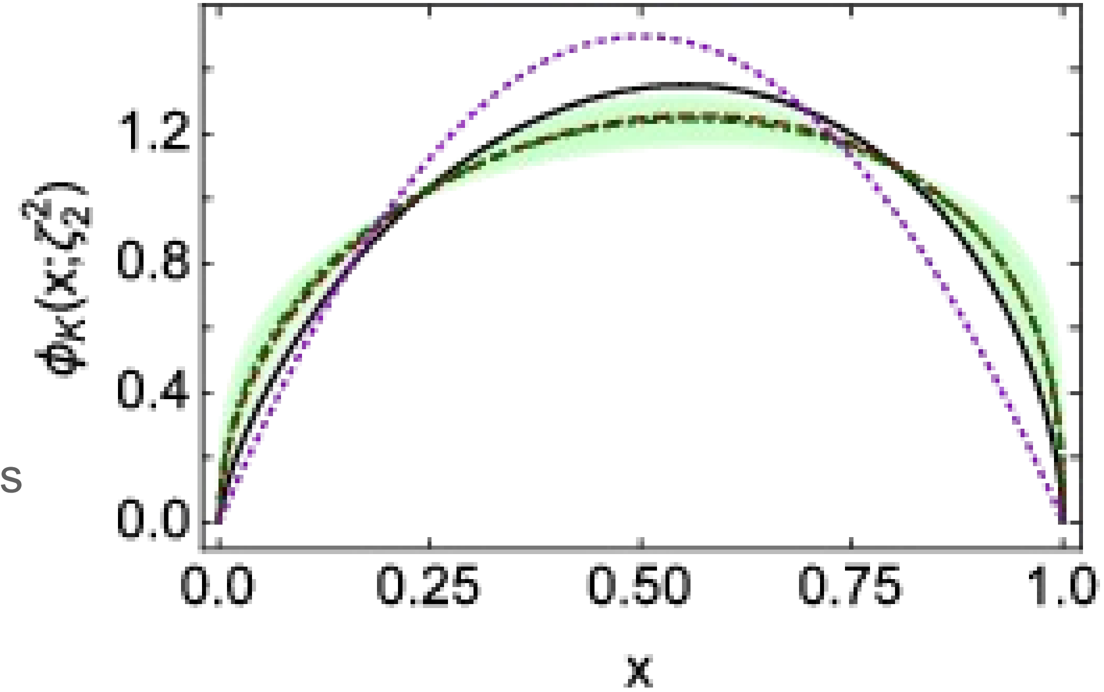


Real-word scale *ζ2*

Flavour symmetry breaking in the kaon parton distribution amplitude, Chao Shi, Lei Chang, Craig D. Roberts, Sebastian M. Schmidt, Peter C. Tandy and Hong-Shi Zong, arXiv:1406:3353 [nucl-th], Phys. Lett. B 738 (2014) pp. 512–518

Lattice-QCD & kaon's valence-quark PDA

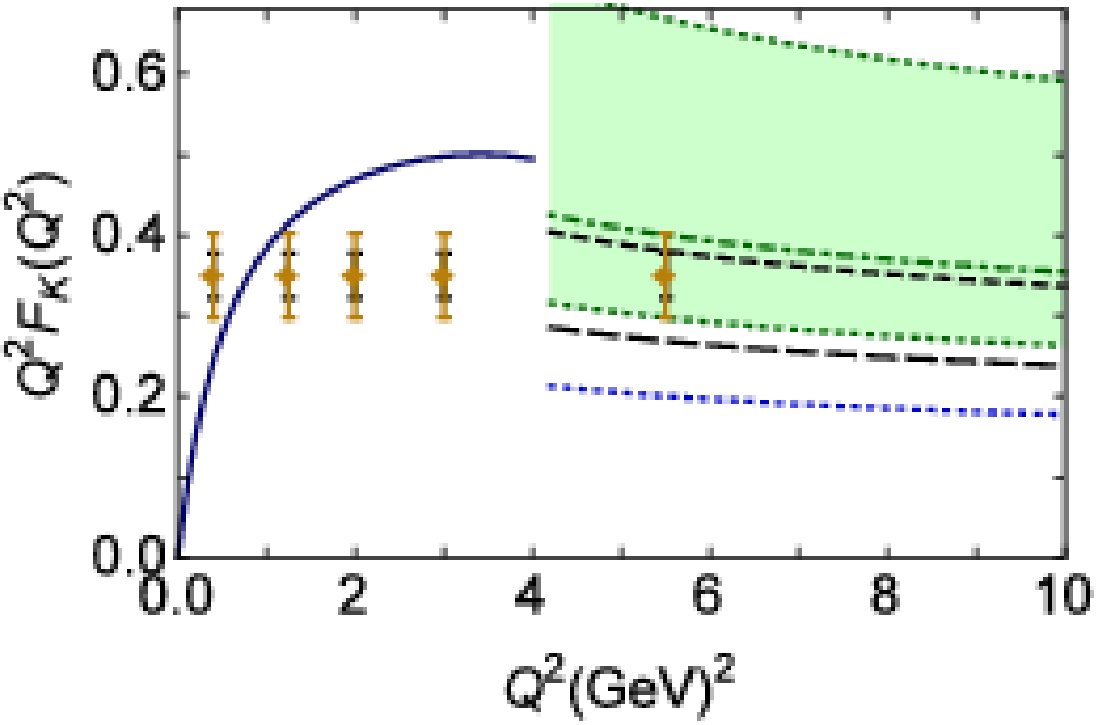
- Isolated dotted curve = conformal QCD
- Green curve & band = result inferred from the single pion moment computed in lattice-QCD
- Black solid and red dashed curves = band of DSE predictions
- Agreement between DSE & lQCD predictions, within errors



The pion: an enigma within the Standard Model
Tanja Horn, Craig D. Roberts: arXiv:1602.04016 [nucl-th]

Kaon's electromagnetic form factor

- Solid blue curve = Maris-Tandy prediction
- Hard-scattering formula
  - Short- and long-dashed curves = DSE prediction for PDA yields result within this area
  - Green band = broad, skewed lQCD PDA
- Skewing is not the issue: 12%-15%, DSE- and lattice-QCD agree
- It's extent of the broadening that generates the uncertainty
- JLab 12 has potential to settle the issue … Meantime, extend *Fπ(Q2)* analysis on entire spacelike domain → *FK(Q2)*



Kaon elastic form factor at all spacelike momenta, Fei Gao et al.

Kaon's electromagnetic form factor

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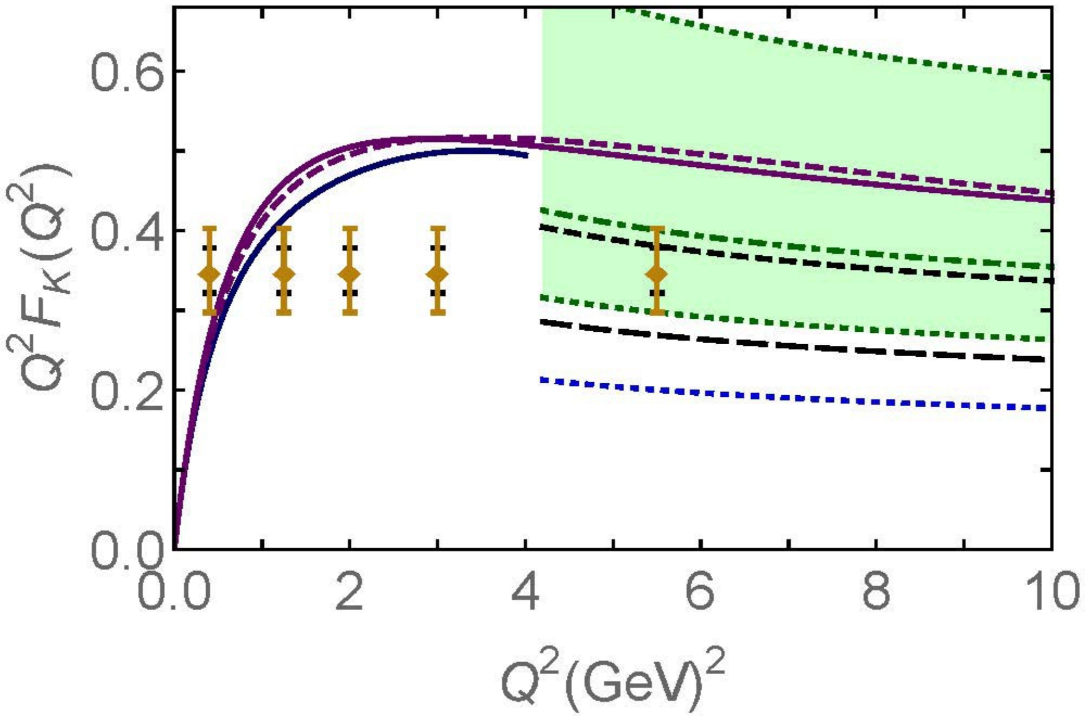
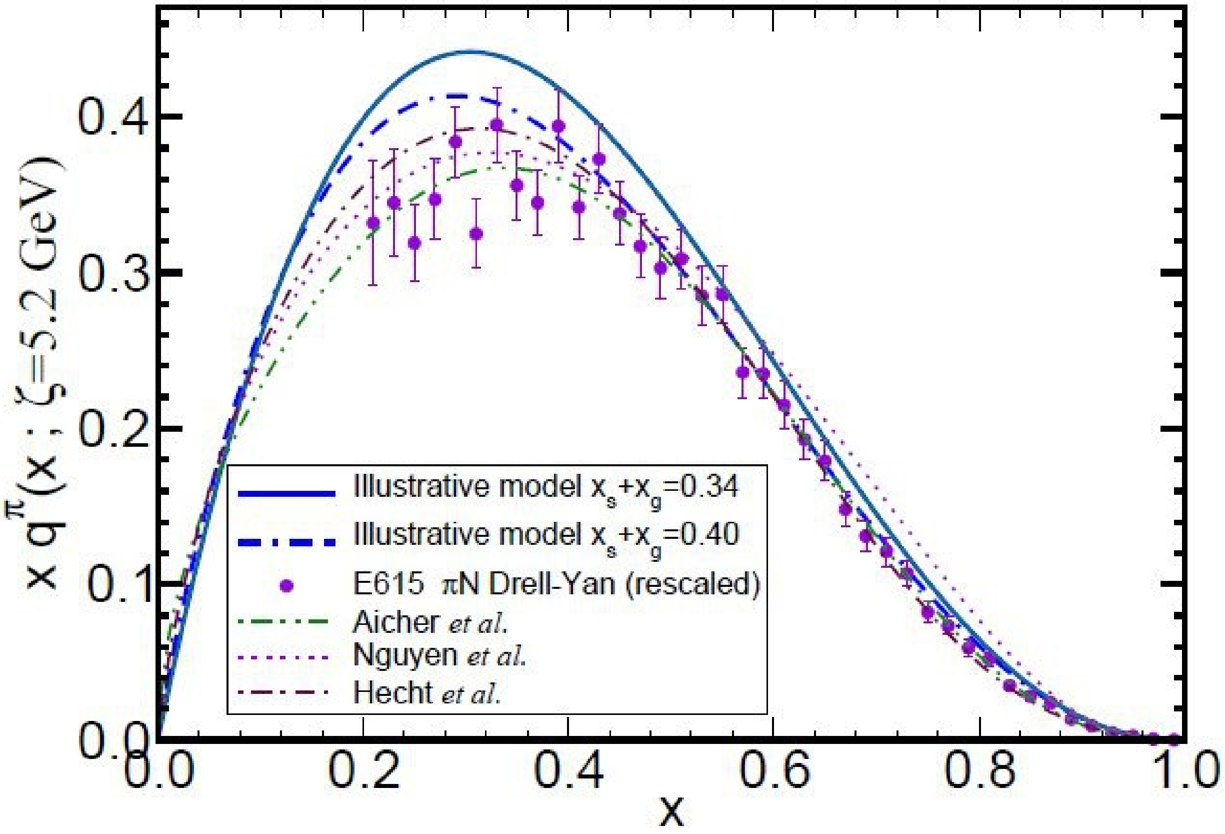




π & *K* Valence-quark Distribution Functions

π & K PDFs

- Experimental data on π & K PDFs obtained in mesonic Drell-Yan scattering from nucleons in heavy nuclei; but it's old: 1980-1989
- Newer data would be welcome:
  - persistent doubts about the Bjorken-$x \simeq 1$ behaviour of the pion's valence-quark PDF
  - single modest-quality measurement of $u_K(x)/u_\pi(x)$ cannot be considered definitive.
- Approved experiment, using tagged DIS at JLab 12, should contribute to a resolution of pion question; and a similar technique might also serve for the kaon.
- Furthermore:
  - new mesonic Drell-Yan measurements at modern facilities could yield valuable information on π and K PDFs,
  - as could two-jet experiments at the large hadron collider;
  - and an EIC would be capable of providing access to π and K PDFs through measurements of forward nucleon structure functions.
- Gribov-Lipatov reciprocity (crossing symmetry) entails connection between PDFs and fragmentation functions on $z \simeq 1$ $(z \geq 0.75)$

$$D_{H/q}(z) \approx z\, q_H(z)$$

Reliable information on meson fragmentation functions is critical if the worldwide programme

Pion’s Valence-Quark Distribution Function

- Hadron PDAs are not directly measurable; but experiments place constraints on their Mellin moments
- One of the earliest predictions of the QCD parton model (1974,1975):

$$q_\pi(x) \sim (1-x)^2$$

- Owing to the validity of factorisation in QCD, $q_\pi(x)$ is directly measurable in πN Drell-Yan experiments
- E615 @ FNAL (Conway:1989fs): leading-order analysis of πN Drell-Yan

$$q_\pi(x) \sim (1-x)^1$$

Apparent *disagreement* with QCD.

Nevertheless used to produce “modern” PDF fits

E615 Controversy

$$q\pi(x) \sim (1\text{-}x)\mathbf{1}$$

Numerous “explanations”

- Nambu – Jona-Lasinio model, translationally invariant regularisaion

  $q\pi(x) \sim (1\text{-}x)\mathbf{0}$,

  which becomes one after evolving from a low resolution scale
- NJL models with a hard cutoff & also some duality arguments:

  $q\pi(x) \sim (1\text{-}x)\mathbf{1}$
- Relativistic constituent quark models:

  $q\pi(x) \sim (1\text{-}x)\mathbf{0\ldots2}$

  depending on the form

  of model wave function
- Instanton-based models

  $q\pi(x) \sim (1\text{-}x)\mathbf{1\ldots2}$

Any power you might like in models possessing no connection with QCD.

What can be learnt from this?

Is QCD threatened?

Basic features of the pion valence-quark distribution function,
Lei Chang, Cédric Mezrag, Hervé Moutarde, Craig D. Roberts, Jose Rodríguez-Quintero and Peter C. Tandy,
arXiv:1406:5450 [nucl-th], Phys. Lett. B **737** (2014)pp. 23–29

Valence-quark distribution functions in the kaon and pion,
Chen Chen, Lei Chang, Craig D. Roberts, Shaolong Wan and Hong-Shi Zong, arXiv:1602.01502 [nucl-th], Phys. Rev. D**93** (2016) 074021/1-11

Valence-quark PDFs within mesons

- Compute PDFs from imaginary part of virtual-photon – pion forward Compton scattering amplitude:
  $\gamma\, \pi \to \gamma\, \pi$
- Handbag diagram is insufficient. Doesn't even preserve global symmetries. Exists a class of leading-twist corrections that remedies this defect ⇒

Projection onto light-front

$$u_V^\pi(x) = N_c \mathrm{tr} \int_{dk} \delta_n^x(k_\eta^\pi)$$

Simila

Partial derivative wrt relative momentum

$$\times\, \partial_{k_\eta^\pi} \left[\Gamma_\pi(k_\eta^\pi, -P_\pi) S(k_\eta^\pi)\right] \Gamma_\pi(k_{\bar\eta}^\pi, P_\pi)\, S(k_{\bar\eta}^\pi)\,,$$

Valence-quark PDFs within mesons

- Formulae guarantee that valence-quark PDFs satisfy, independent of model details:

$$\langle x\rangle_u^0 = \int_0^1 dx\, x\, u_V^0(x) = \frac{1}{2}$$

$$\int_0^1 dx\, x[u_V^K(x) + \bar{s}_V^K(x)] = 1$$

- Ensure, too, that at an hadronic scale ζ ≈ 0.5 GeV

  qVM(x ≃ 1) ∝ (1-x)2n

  in any theory with *(1/k2)n* vector-boson exchange interaction

Valence-quark PDFs within mesons

- Algebraic analysis
  - *Δf(k2)=1/[k2+Mf2]*
  - *MR=Mus2/[Mu+Ms]*
  - *Mus2 = Mu Ms*
  - *Mu , Ms* = dressed-quark mass-scales: *Mu*=0.4GeV and *Ms=1.2Mu* from most sophisticated DSE analyses
  - β = 1 is kaon asymmetry parameter, fixed to reproduce best DSE result for first moment of kaon PDA
  - *Λπ,K* = widths of meson Bethe-Salpeter amplitudes: chosen to ensure *fπ,K* reproduce empirical values

$$S_f(k) = [-i\gamma\cdot k + M_f]\Delta_{M_f}(k^2)\,,$$

$$\Gamma_\pi(k_{\bar\eta/\eta};\pm P) = i\gamma_5\frac{M_u}{n_\pi}\frac{3}{4}\int dz \times (1-z^2)M_u^2\Delta_{\Lambda_\pi}(k_z^2)\,,$$

$$\Gamma_K(k_{\bar\eta/\eta};\pm P) = i\gamma_5\frac{M_R}{n_K}\frac{3}{4}\int dz \times (1-z^2)(1+\beta z)M_{us}^2\Delta_{\Lambda_K}(k_z^2)$$

Could use sophisticated DSE input, like Tandy et al. *arXiv:1102.2448 [nucl-th]; but, as demonstrated in arXiv:1602.01502 [nucl-th], nothing material is gained*

Predictions of algebraic framework are indistinguishable and provide additional, novel insights

Valence-quark PDFs within mesons

- *x=1* values of ratios of PDFs are invariant under DGLAP evolution

  Consequently, they’re a scale invariant discriminator between competing pictures of hadron structure

$$\left.\frac{u_V^K(x)}{u_V^\pi(x)}\right|_{x\to1} = 0.37\,, \qquad \left.\frac{u_V^\pi(x)}{\bar{s}_V^K(x)}\right|_{x\to1} = 0.29$$

- On the other hand

$$\lim_{x\to0}\frac{u^K(x;\zeta)}{u^\pi(x;\zeta)} \overset{\Lambda_{\rm QCD}/\zeta\simeq0}{\longrightarrow} 1$$

  Owes to inexorable gr… -quark content driven by pQCD splitting mechan…

  Analogous to the convergence of all meson PDAs to conformal form as *ΛQCD/ζ → 0*

Building realistic distributions

- To sensibly evolve PDFs, must include sea & glue at hadronic scale, ζH
- Pion distribution at ζH
  - DSE prediction, following from analysis of leptonic decay: π contains 5% sea
  - Assume GRV analysis of πN Drell-Yan is reliable, then 30% of π momentum carried by glue

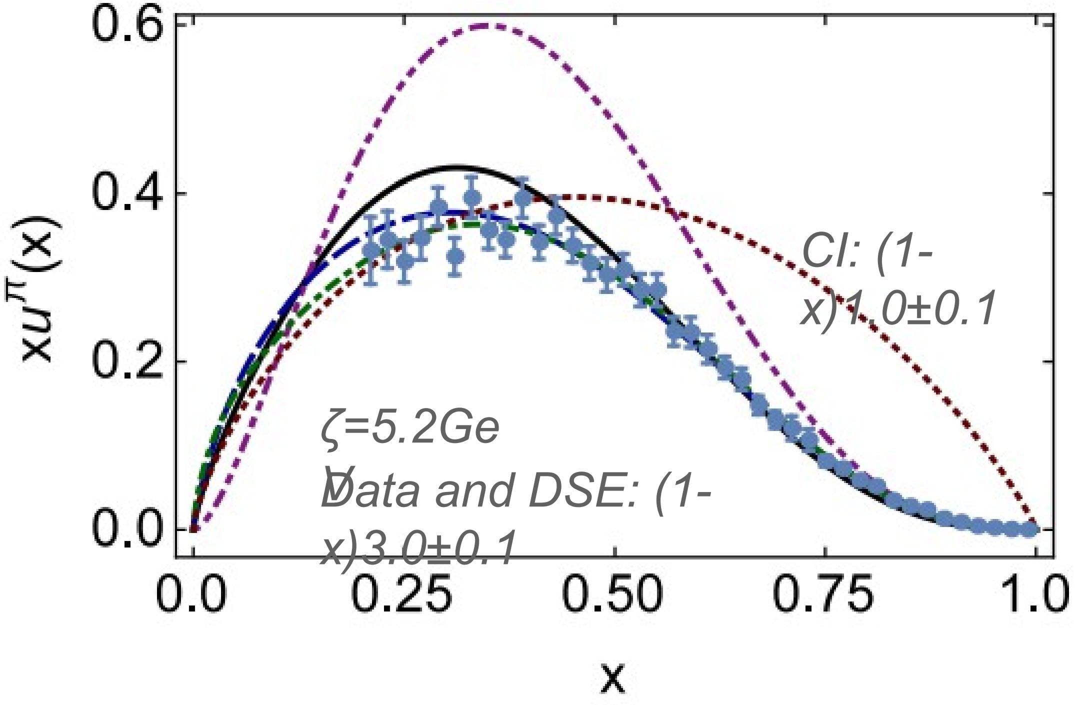
  Adopting standard PDF parametrisations, this additional information is sufficient to completely fix realistic $u\pi(x; \zeta H) = valence + sea + glue$

- Kaon distribution at ζH
  - Owing to heavier mass of intermediate states that can introduce sea-quarks, safe to assume sea-quark content of kaon is effectively zero
  - Treat momentum fraction carried by glue as a parameter to be used in understanding $uK(x)/u\pi(x)$

    … owing to heavier mass of s-quark, expect $\langle x\rangle gK < \langle x\rangle g\pi$;

    … but how much less?

Pion PDF



CI: (1-x)1.0±0.1

ζ=5.2GeV

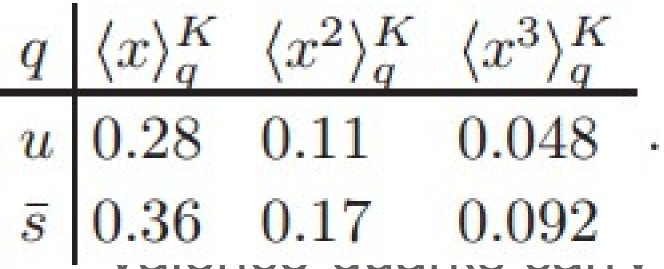
Data and DSE: (1-x)3.0±0.1

FIG. 3. $xu^\pi(x;\zeta_{5.2})$. Solid (black) curve, our prediction, expressed in Eqs. (32), (33); dot-dot-dashed (purple) curve, result obtained when sea-quark and gluon contributions are neglected at ζ_H, *i.e.* using $u_V^\pi(x)$ from Eqs. (14), (17); dashed (blue) curve first DSE prediction [38]; and data, Ref. [4], rescaled according to the reanalysis described in Ref. [40], from which the dot-dashed (green) curve is drawn. The dotted (red) curve is the result obtained using a Poincaré-covariant regularisation of a contact interaction, Eq. (36).

- Purple dot-dot-dash = prediction at *ζH*
- Data = modern reappraisal of E615: NLO analysis plus soft-gluon resummation (ASV)
- Solid black curve, prediction evolved to *ζ*=5.2GeV, the scale associated with the experiments
- Blue dashed curve = first DSE prediction, in 2000 (*ζ*=5.2GeV)
- Dotted red curve = result obtained with momentum-independent gluon exchange (contact interaction, *ζ*=5.2GeV)

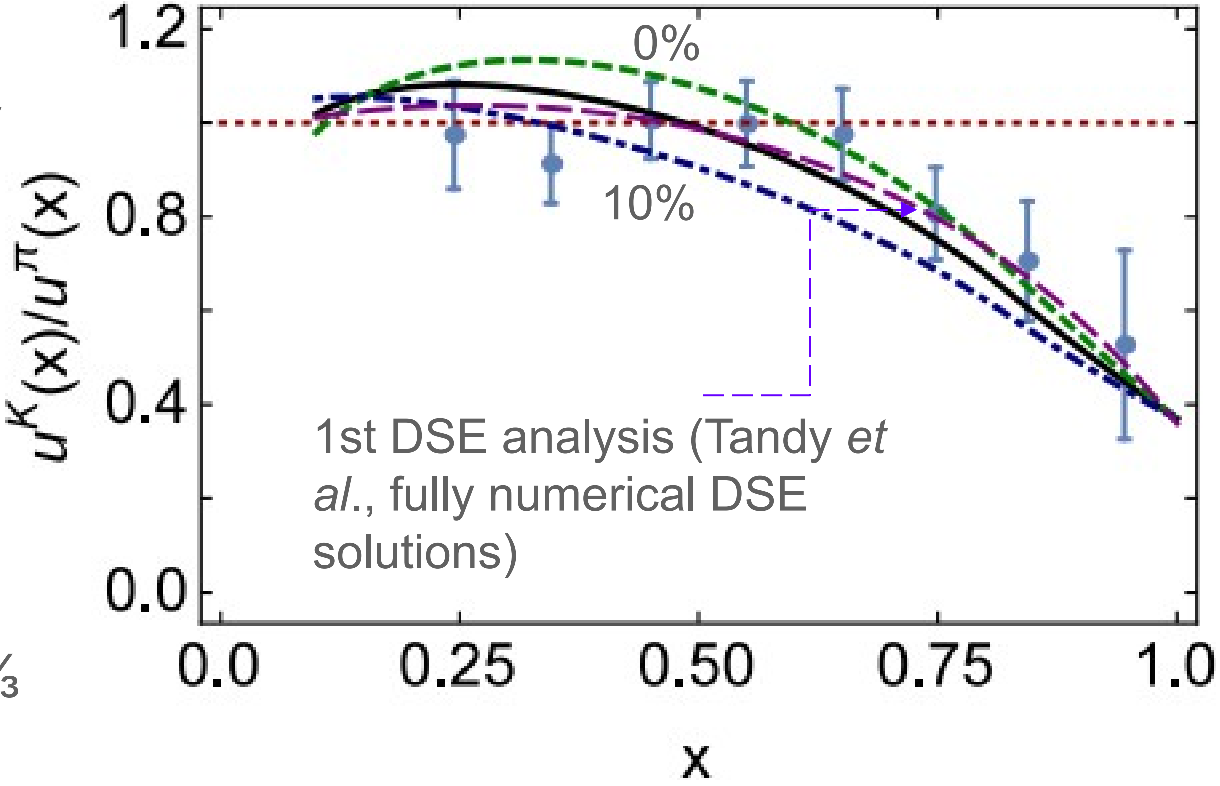
Gluon content of kaon

- $\langle x\rangle gK(\zeta H) = 0.05 \pm 0.05$
  ⇒ Valence quarks carry 95% of kaon's momentum at *ζH*
- Evolved to *ζ2*



q	$\langle x\rangle_q^K$	$\langle x^2\rangle_q^K$	$\langle x^3\rangle_q^K$
u	0.28	0.11	0.048
$\bar{s}$	0.36	0.17	0.092

Valence quarks carry ⅔ of kaon's light-front momentum



1st DSE analysis (Tandy *et al.*, fully numerical DSE solutions)

π & K PDFs

- Dressed-quark basis and symmetry-preserving (beyond-handbag) expressions used to analyse π & K valence-quark PDFs … guarantee that at hadronic scale

  $qV(x;\zeta H) \propto (1-x)2$ on $x \simeq 1$
- Flavour-dependence of DCSB modulates the strength of SU(3)-flavour symmetry breaking in meson PDFs, as it does in every other nonperturbative quantity
- At ζH:
  - valence dressed-quarks carry roughly two-thirds of pion's light-front momentum, with the bulk of the remainder carried by glue … sea-quarks carry roughly 5%
  - contrast, valence dressed-quarks carry approximately 95% of the kaon's light-front momentum, with the remainder lying in the gluon distribution … sea-quarks carry ≃ 0 %
    - heavier *s*-quarks radiate soft gluons less readily than lighter quarks and momentum conservation subsequently constrains gluons associated with the kaon's *u*-quark

π & *K* PDFs

- Evolving distributions to scale characteristic of meson-nucleon Drell-Yan experiments, $\zeta=5.2$ GeV
  - extant data reproduced
    - $x \simeq 1$: $q(x) \propto (1-x)3$
  - ratio $uK(x)/u\pi(x)$ explained by vastly different gluon content of *π* & *K*
- Distributions evolved the distributions to $\zeta 2 = 2$ GeV, which is typically used in numerical simulations of lattice-regularised QCD:
  - Valence-quarks carry roughly half the pion’s light-front momentum but two-thirds of the kaon’s momentum.
- *Next steps*
  - *Do not anticipate any improvement over these results using continuum methods in QCD*
    - *Can lQCD say anything novel? (As yet, nothing from lattice on kaon PDF)*
  - *Empirical verification of the predictions is essential (equivalent to verifying hard-scattering formulae)*
  - *Extend analysis to GPDs, TMDs and Fragmentation Functions*

π & *K* PDFs

- Marked differences between *π* & *K* gluon content
  - *ζH*:
    - One-third of pion’s light-front momentum carried by glue
    - One-twentieth of the kaon’s light-front momentum lies with glue
  - *ζ*22 = 4 GeV2
    - Glue carries half of pion’s momentum and two-thirds of kaon’s momentum
  - Evident in differences between large-*x* behaviour of valence-quark distributions in these two mesons
- Signal of Nambu-Goldstone boson character of *π*
  - Nearly complete cancellation between
    one-particle dressing and binding attracti

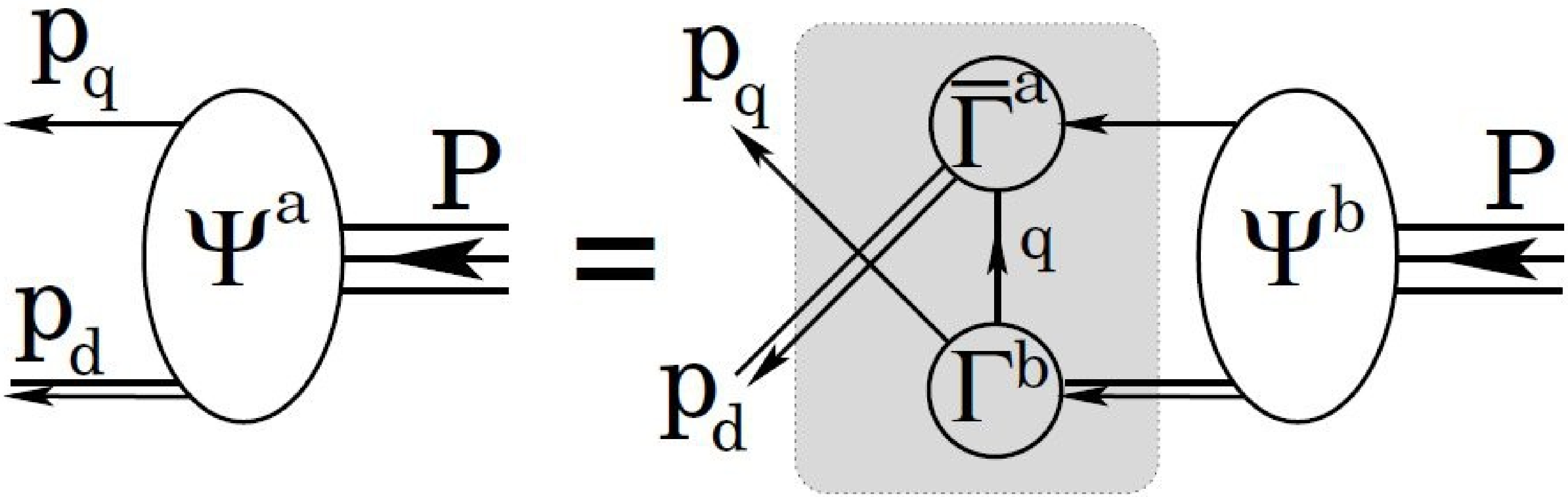
    in this almost massless pseudoscalar sys

    2 M + U ≈ 0

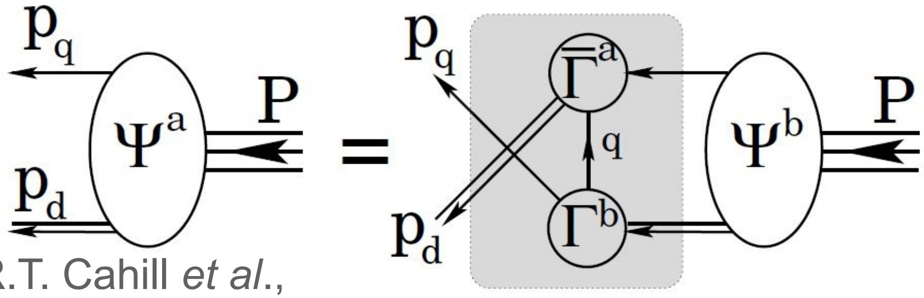
π & K PDFs

- Existing textbook description of Goldstone's theorem via pointlike modes is old-fashioned, outdated and simplistic
- The appearance of Nambu-Goldstone modes in the Standard Model is far more interesting
  - Nambu-Goldstone modes are nonpointlike!
  - Intimately connected with origin of mass!
  - Quite probably inseparable from expression of confinement!
- Difference between gluon content is measurable … using well-designed EIC
- Write a definitive new chapter in future textbooks on the Standard Model

Electron Ion Collider:
The Next QCD Frontier



Spectrum & Structure of Baryons



.T. Cahill *et al.*,
ustral. J. Phys. 42 (1989) 129-145

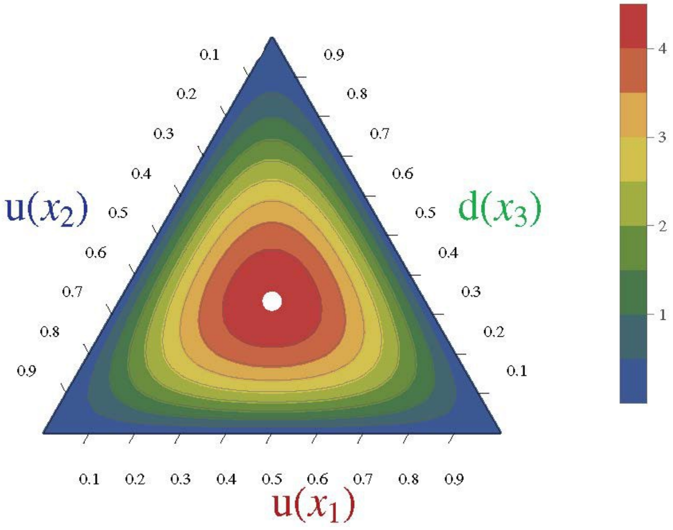
Baryon Structure

- Poincaré covariant Faddeev equation sums all possible exchanges and interactions that can take place between three dressed-quarks
- Confinement and DCSB are readily expressed
- ***Prediction***: owing to *DCSB in QCD, strong diquark correlations exist within baryons*
- Diquark correlations are not pointlike
  - Typically, *r0+ ~ rπ* & *r1+ ~ rρ* (actually 10% larger)
  - They have soft form factors

Cédric Mezrag, *et al*. ANL

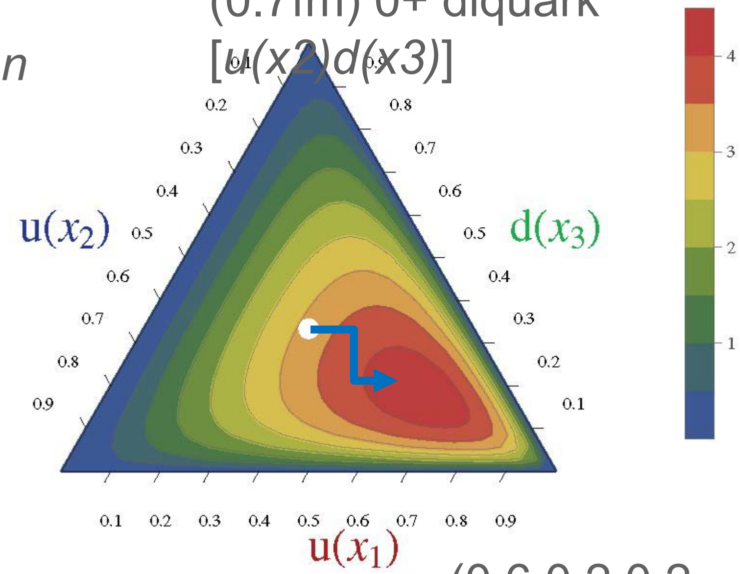
Nucleon Parton Distribution Amplitudes

- Computations underway. First results available.



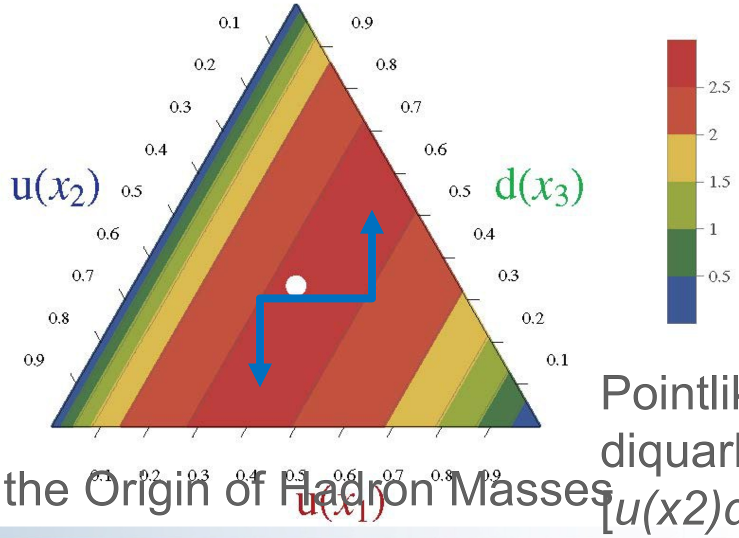
Diquark clustering skews the distribution toward the dressed-quark bystander, which therefore carries more of the proton's light-front momentum

Realistic, finite size (0.7fm) 0+ diquark [*u(x2)d(x3)*]



(0.6,0.2,0.2)

conformal limit:
120 x1x2x3
〈*xi*〉 =⅓ … peak of the distribution



Pointlike 0+ diquark
[*u(x2)d(x3)*]

Light-cone distribution amplitudes of the nucleon and negative parity nucleon resonances from lattice QCD
V. M. Braun *et al*., Phys. Rev. D 89 (2014) 094511

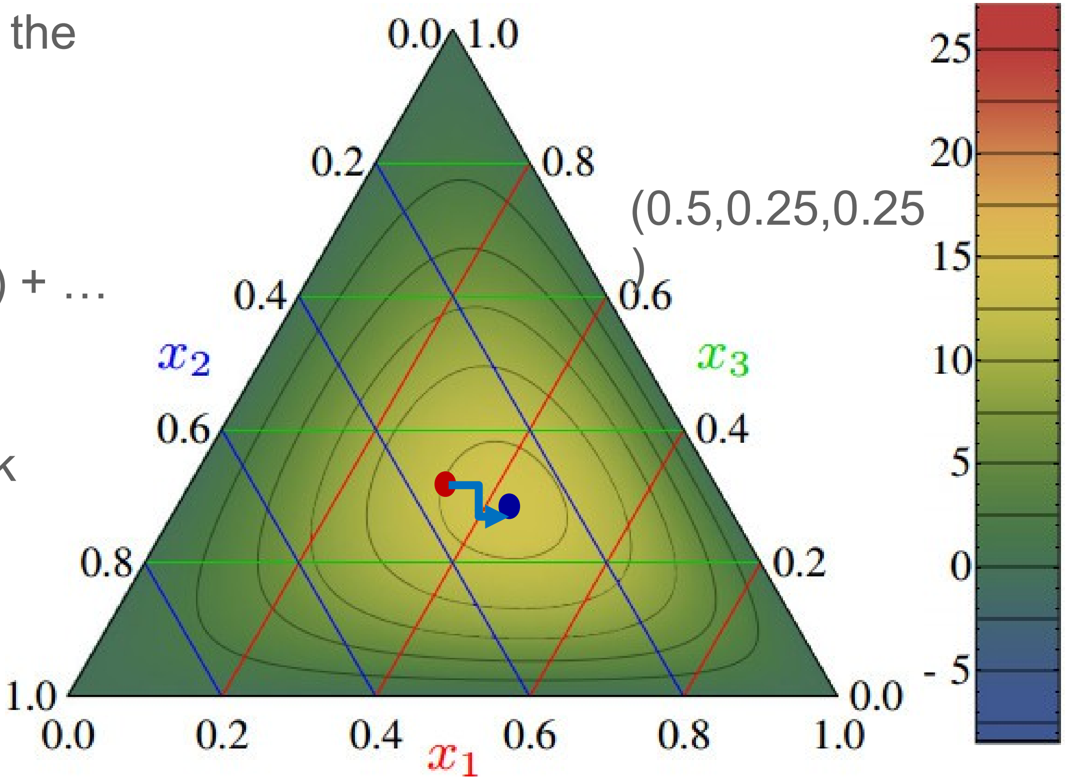
Light-cone distribution amplitudes of the baryon octet
G. S. Bali *et al*. JHEP 1602 (2016) 070

Nucleon PDAs & lQCD

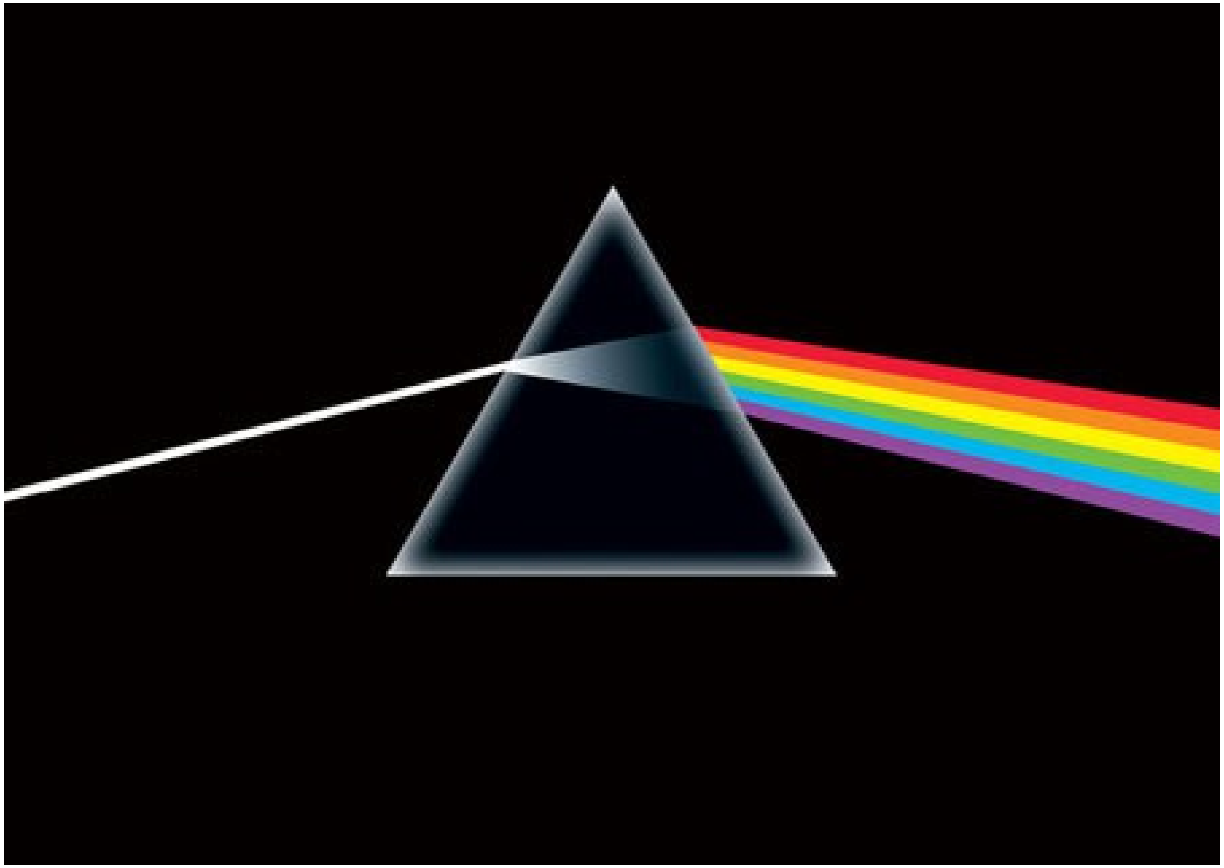
- First lQCD results for n=0, 1 moments of the leading twist PDA of the nucleon are available
- Used to constrain strength (a_{11}) of the leading-order term in a conformal expansion of the nucleon's PDA:

$\Phi(x_1,x_2,x_3)$
$= 120\, x_1 x_2 x_3\, [\, 1 + a_{11} P_{11}(x_1,x_2,x_3) + \ldots \,]$

- Shift in location of central peak is consistent with existence of diquark correlations within the nucleon



(0.5,0.25,0.25)



Far valence domain

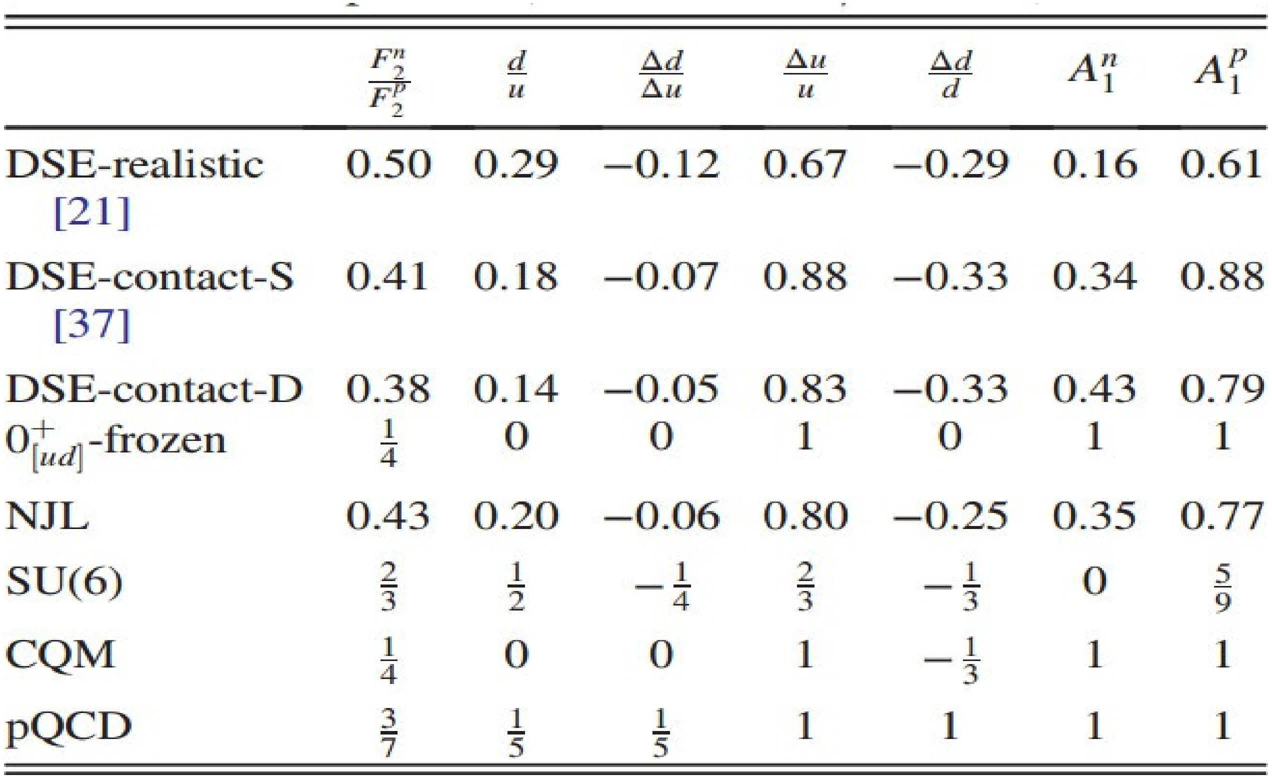
x≃1

Nucleon spin structure at very high-x
Craig D. Roberts, Roy J. Holt and Sebastian M. Schmidt
arXiv:1308.1236 [nucl-th], Phys. Lett. B 727 (2013) pp. 249–254
Contact-interaction Faddeev equation and, inter alia, proton tensor charges, Shu-Sheng Xu et al., *arXiv:1509.03311 [nucl-th], Phys. Rev. D 92 (2015) 114034/1-21*

Nucleon PDFs at large Bjorken-x

- Correlations between dressed-quarks within the proton have an enormous impact on nucleon flavor & spin structure

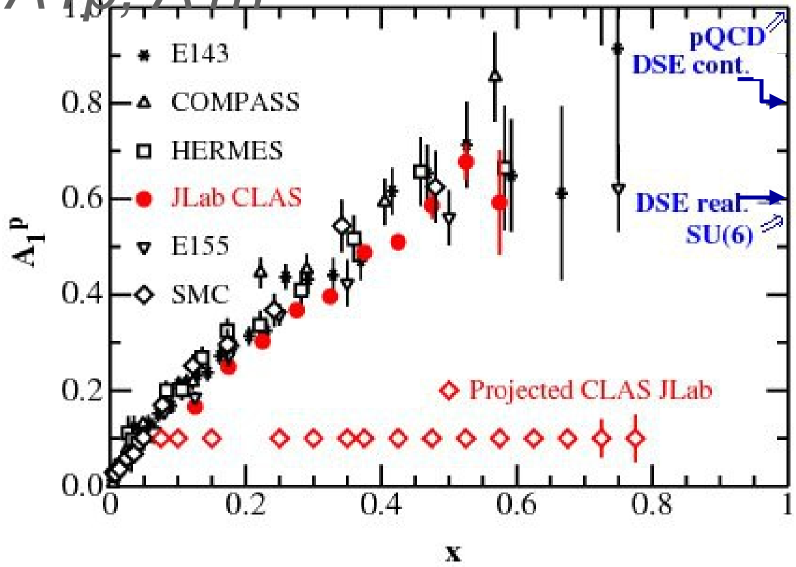
No two frameworks make the same predictions for F2n/F2p, A1p, A1n

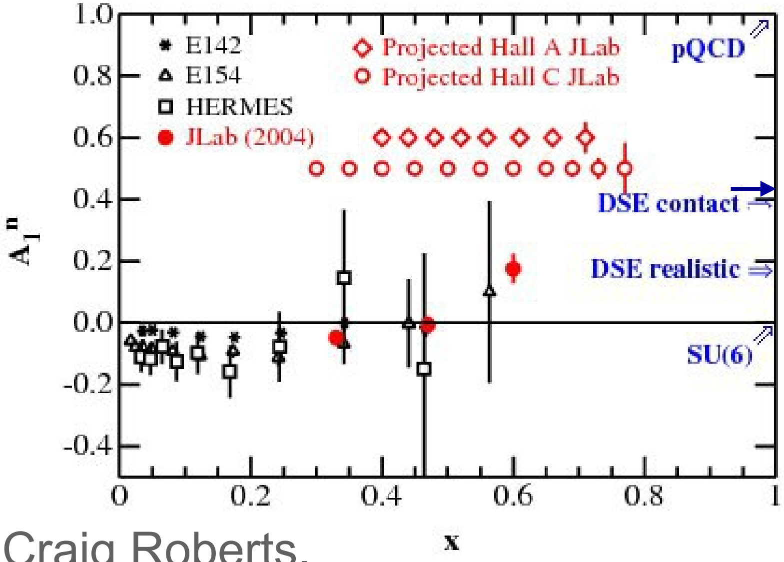


	$\frac{F_2^n}{F_2^p}$	$\frac{d}{u}$	$\frac{\Delta d}{\Delta u}$	$\frac{\Delta u}{u}$	$\frac{\Delta d}{d}$	A_1^n	A_1^p
DSE-realistic [21]	0.50	0.29	−0.12	0.67	−0.29	0.16	0.61
DSE-contact-S [37]	0.41	0.18	−0.07	0.88	−0.33	0.34	0.88
DSE-contact-D	0.38	0.14	−0.05	0.83	−0.33	0.43	0.79
$0^+_{[ud]}$-frozen	$\frac{1}{4}$	0	0	1	0	1	1
NJL	0.43	0.20	−0.06	0.80	−0.25	0.35	0.77
SU(6)	$\frac{2}{3}$	$\frac{1}{2}$	$-\frac{1}{4}$	$\frac{2}{3}$	$-\frac{1}{3}$	0	$\frac{5}{9}$
CQM	$\frac{1}{4}$	0	0	1	$-\frac{1}{3}$	1	1
pQCD	$\frac{3}{7}$	$\frac{1}{5}$	$\frac{1}{5}$	1	1	1	1

No two frameworks make the same predictions for F2n/F2p, A1p, A1n

Quark helicity at large Bjorken-x

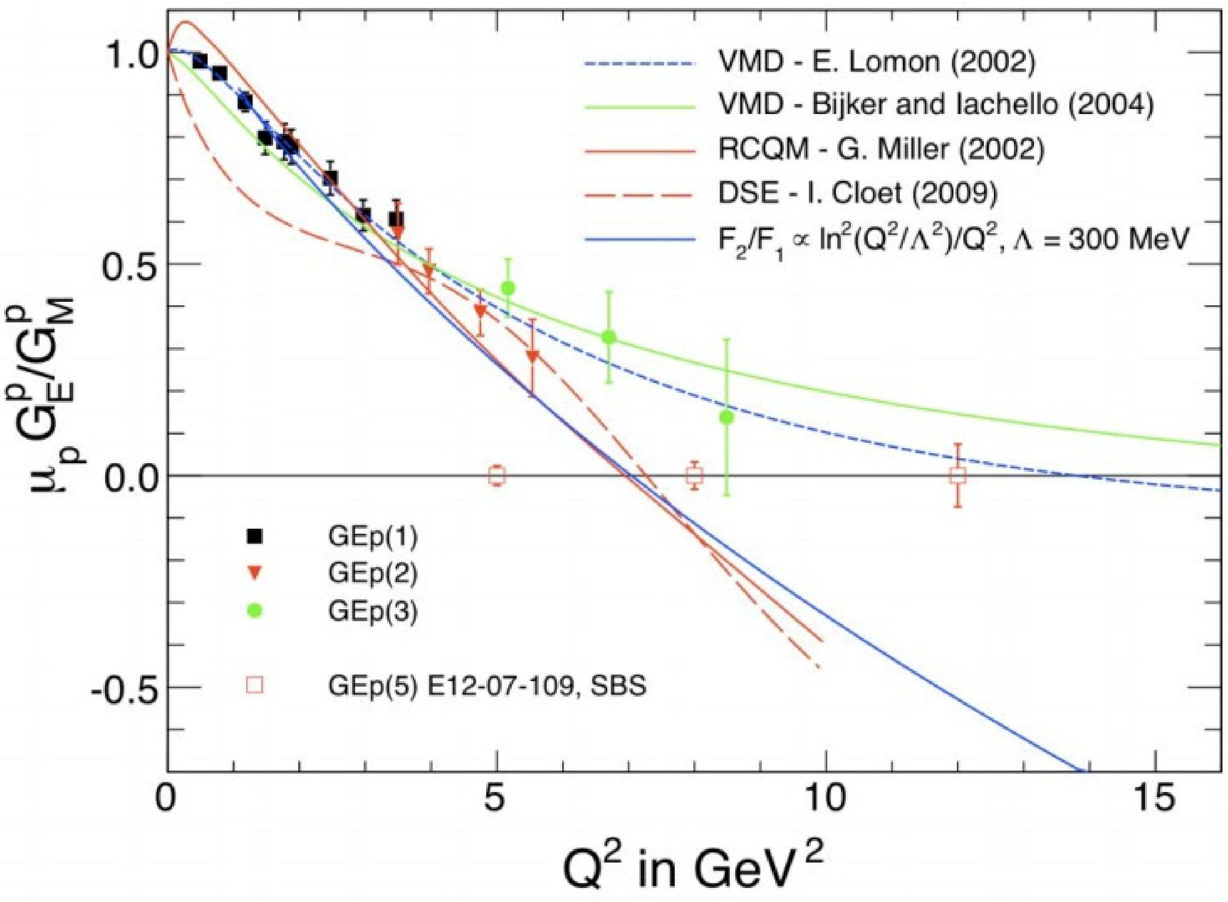




- Existing data cannot distinguish between modern pictures of nucleon structure
- Empirical results for nucleon longitudinal spin asymmetries on *x* ≃ *1* promise to add greatly to our capacity for discriminating between contemporary pictures of nucleon structure

Electron Ion Collider: The Next QCD Frontier

GEp5 Projected results

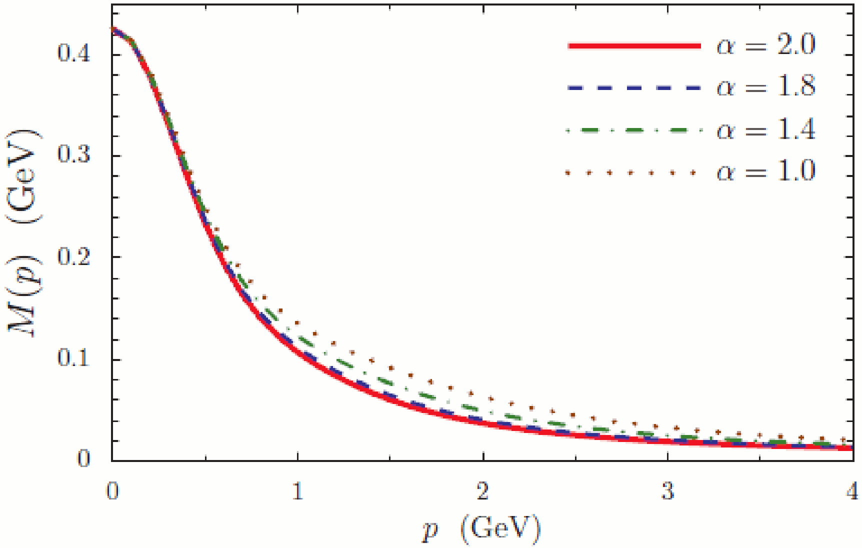


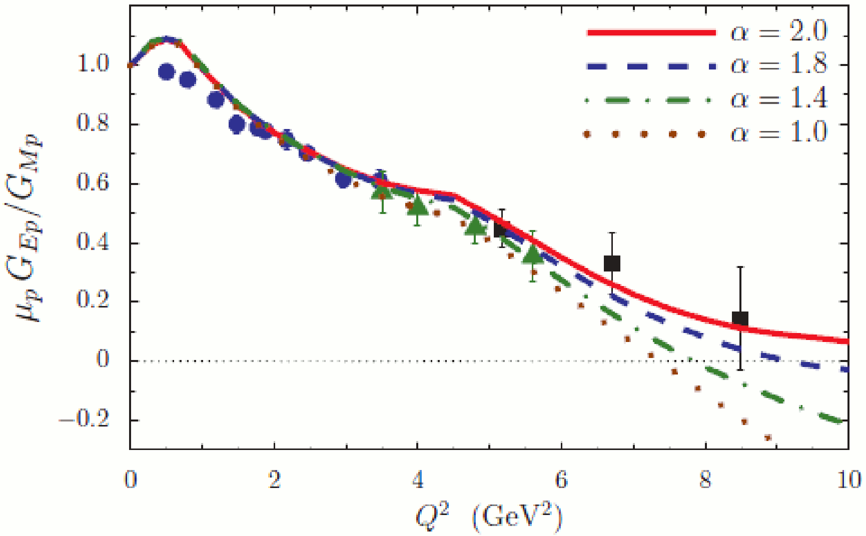
Nucleon Elastic FFs

I.C. Cloët, C.D. Roberts, A.W. Thomas: Revealing dressed-quarks via the proton's charge distribution, arXiv:1304.0855 [nucl-th], Phys. Rev. Lett. **111** (2013) 101803

Visible Impacts of DCSB

$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$

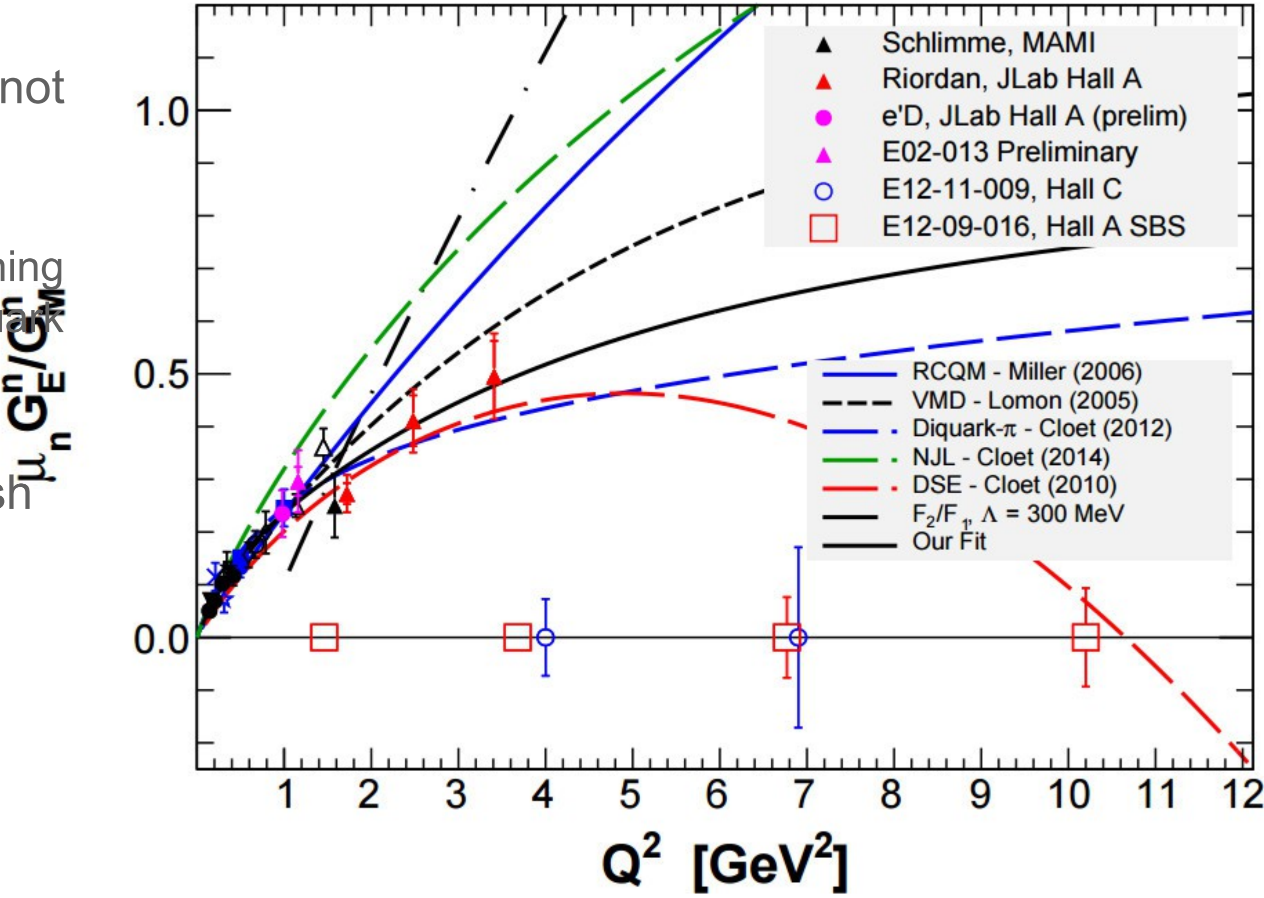




- Possible existence and location of a zero in the ratio of proton elastic form factors *[μpGEp(Q2)/GMp(Q2)]* *are* a direct measure of the rate at which dressed-quarks become partons again, *i.e*. character of strong interactions in the Standard Model.

GEn

- Numerous calculations on this figure; but only one viable prediction
- DSE result (2008/2010) is not fitted to any data
    - Predicts zero in *GEn*
    - Owes to presence of running quark-mass & strong diquark correlations
    - Verifiable at JLab12
- *GEn* promises to be a harsh discriminator between descriptions of nucleon structure



Seamus Riordan, ECT, April 2016*

γ N → Resonance

- Prediction and measurement of ground-state elastic form factors is essential to increasing our understanding of strong-interaction
- However, alone, it is insufficient to chart the infrared behaviour of the strong interaction
  - the hydrogen ground-state didn't give us QED
- There are numerous nucleon → resonance transition form factors.
  - The challenge of mapping their *Q2*-dependence provides a vast array of novel ways to probe the infrared behaviour of the strong interaction, including the environment and energy sensitivity of correlations

Baryons

Nucleon and Δ elastic and transition form factors
Jorge Segovia *et al*., arXiv:1408.2919 [nucl-th], Few Body Syst. **55** (2014) pp. 1185-1222

Completing the picture of the Roper resonance
Jorge Segovia *et al*. arXiv:1504.04386 [nucl-th], Phys. Rev. Lett. **115** (2015) 171801

Dissecting nucleon transition electromagnetic form factors
Jorge Segovia and Craig D. Roberts, arXiv:1607.04405 [nucl-th], Phys. Rev. C (Rapid Comm.) *in press*

- Critical issues:
  - is there an environment sensitivity of DCSB and the dressed-quark mass function?
  - are quark-quark correlations an essential element in the structure of all baryons?
- Existing feedback between experiment and theory indicates that there is no environment sensitivity for the nucleon, Δ-baryon and Roper resonance:
  - DCSB in these systems is expressed in ways that can readily be predicted once its manifestation is understood in the pion, and this includes the generation of diquark correlations with the same character in each of these baryons.

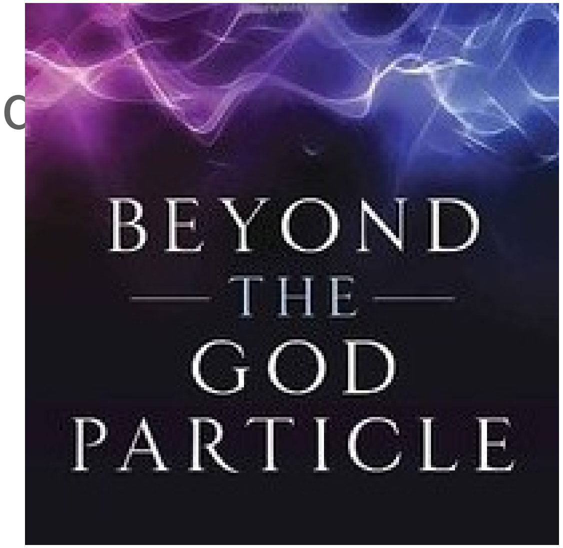
Epilogue

Epilogue

- LHC has NOT found the “God Particle” because the Higgs boson is NOT the origin of mass
  - Higgs-boson only produces a little bit of mass
  - Higgs-generated mass-scales explain neither the proton’s mass nor the pion’s (*near*-)masslessness
  - Hence LHC has, as yet, taught us very little about the origin, structure and nature of the nuclei whose existence support the Cosmos
- Strong interaction sector of the Standard Mod
  i.e. QCD, is the key to understanding the

  origin, existence and properties

  of (almost) all known matter



BEYOND
THE
GOD
PARTICLE

Epilogue

- Conformal anomaly ... *gluons and quarks acquire momentum-dependent masses* … values are large in the infrared *mg ∝ 500* MeV & *Mq ∝ 350* MeV … underlies DCSB; and has numerous observable consequences
- Appearance of *conformal anomaly* is not intrinsically tied to non-Abelian nature of QCD; but its *strength is fixed by gluon self-interactions*
- QCD is plausibly a mathematically well-defined quantum field theory,

  The only one we've ever produced
  - Asymptotic freedom means … ultraviolet behaviour of QCD is controllable
  - Dynamically generated masses for gluons and quarks means that QCD dynamically generates its own infrared cutoffs
  - Consequently, it is a worthwhile paradigm for developing Beyond-SM theories
- Universe with light quarks ⇒ confinement is a dynamical phenomenon

  … *no linear potentials, no tower of linear, nonintersecting "Regge" trajectories … no QCD-connected analysis has every produced them*

Epilogue

- Origin and distribution of mass depend on the observer’s preferred frame of reference and scale
- At scale typical of contemporary and planned experiments, DCSB paradigm is the best way to explicate and understand the associated, emerging phenomena. Numerous verifiable predictions accessible
  - Hadron form factors
  - Hadron PDFs
  - Extends to hadron GPDs and TMDs
- What can experiments at an EIC add to this?
  - Current focus is on low-*x*
  - Gluons dominate … mass generation in the gluon sector, too, is essentially nonperturbative … must affect potential for gluon saturation; but how?
  - Intelligent design will ensure valence-quark region is also accessible
- Ability to compute valence-quark PDAs and PDFs has provided many new insights … must now begin to do the same for sea-quarks *and gluo*

$$\tau_1^{qk} = a_1 \frac{\Delta_B^{qk}}{q^2+k^2} \qquad \tau_3^{qk} = -a_3 2\Delta_A^{qk} ,$$

$$\tau_4^{qk} = a_4 \frac{4\Delta_B^{qk}}{t^{\rm T}\cdot t^{\rm T}} , \; \tau_5^{qk} = a_5 \Delta_B^{qk} ,$$

$$\tau_8^{qk} = a_8 \Delta_A^{qk} ,$$

$$T_\nu^1 = \tfrac{i}{2} t_\nu^{\rm T} , \qquad T_\nu^3 = \gamma_\nu^{\rm T} ,$$

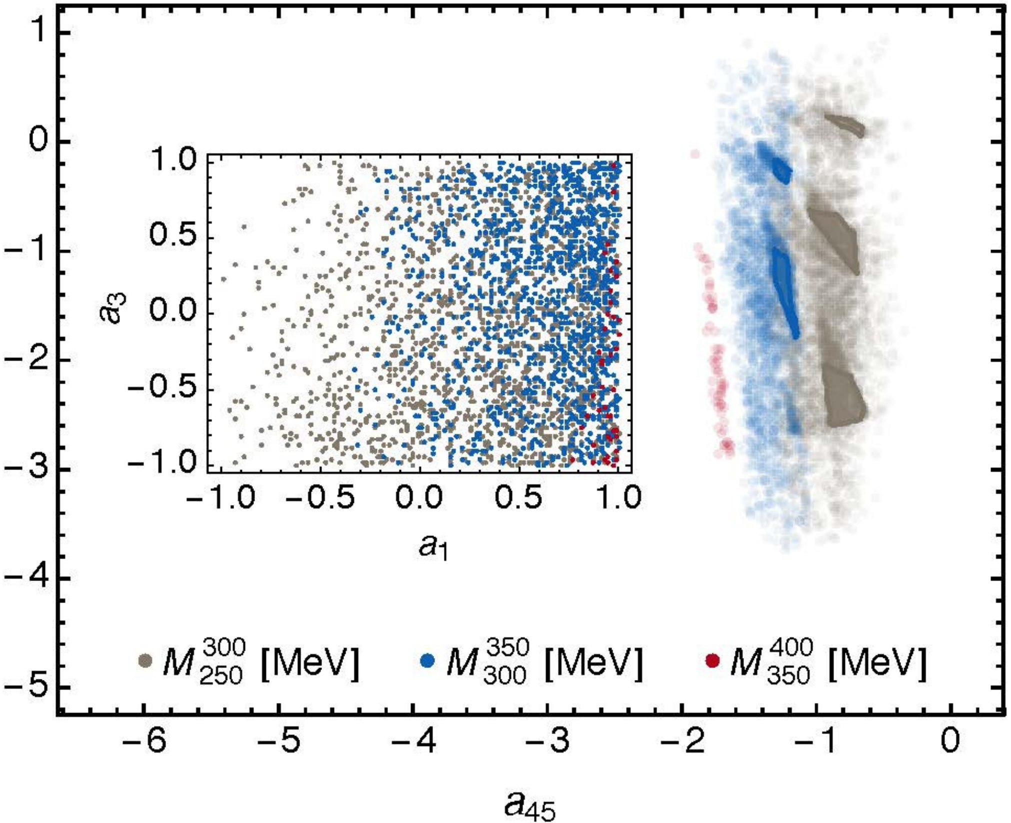
$$T_\nu^4 = -i T_\nu^1 \sigma_{\alpha\beta} q_\alpha k_\beta , \; T_\nu^5 = \sigma_{\nu\rho} p_\rho ,$$

$$T_\nu^8 = q_\nu \gamma\cdot k - k_\nu \gamma\cdot q + i\gamma_\nu \sigma_{\alpha\beta} q_\alpha k_\beta , \qquad \text{(A1)}$$

- Even a small selection of observables places extremely tight bounds on the domain of acceptable, realistic vertex *Ansätze*
- Meson spectrum ⇒ *a2,6,7=0*

  (Sixue Qin *et al.*)
- In **R**4 … subset of (almost)

$$\mathbb{G}_4 \subset \{(a_1, a_3, a_{\hat{4}\hat{5}}, a_8) \,|\, a_1 \in [-0.5, 1],$$
$$a_3 \in [-1, 1], a_{\hat{4}\hat{5}} \in [-2, -0.4] \,, a_8 \in [-4, 1]\}$$



Dressed-gluon-quark vertex

Gap equation only “feels” a45=a4-3a5

arXiv:1602.01502 [nucl-th]

Valence-quark PDFs within mesons

- Pointwise results obtained via reconstruction from (arbitrarily many) computed PDF moments
- Peak in kaon PDFs shifted 17% away from x=½,

  i.e. scale of flavour symmetry breaking is set by DCSB (*Ms/Mu*=1.2), here as in all other nonperturbative quantities
- *[uVK(x)+sVK(x)]/2* must be symmetric, owing to momentum sum rule.

  Similar but not identical to *uVπ(x)*

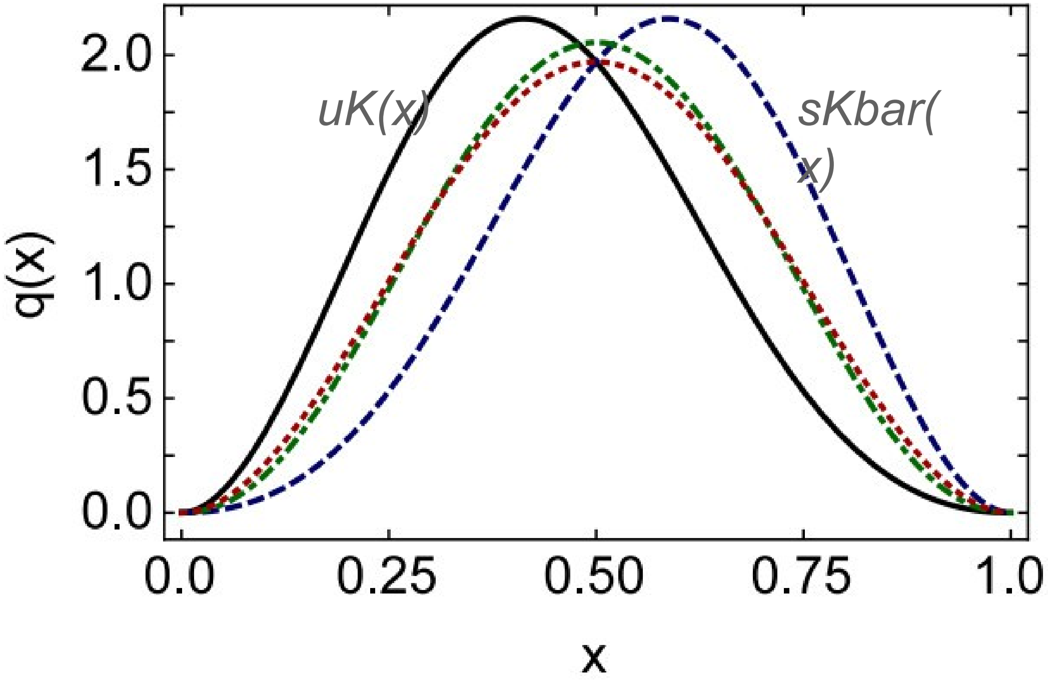
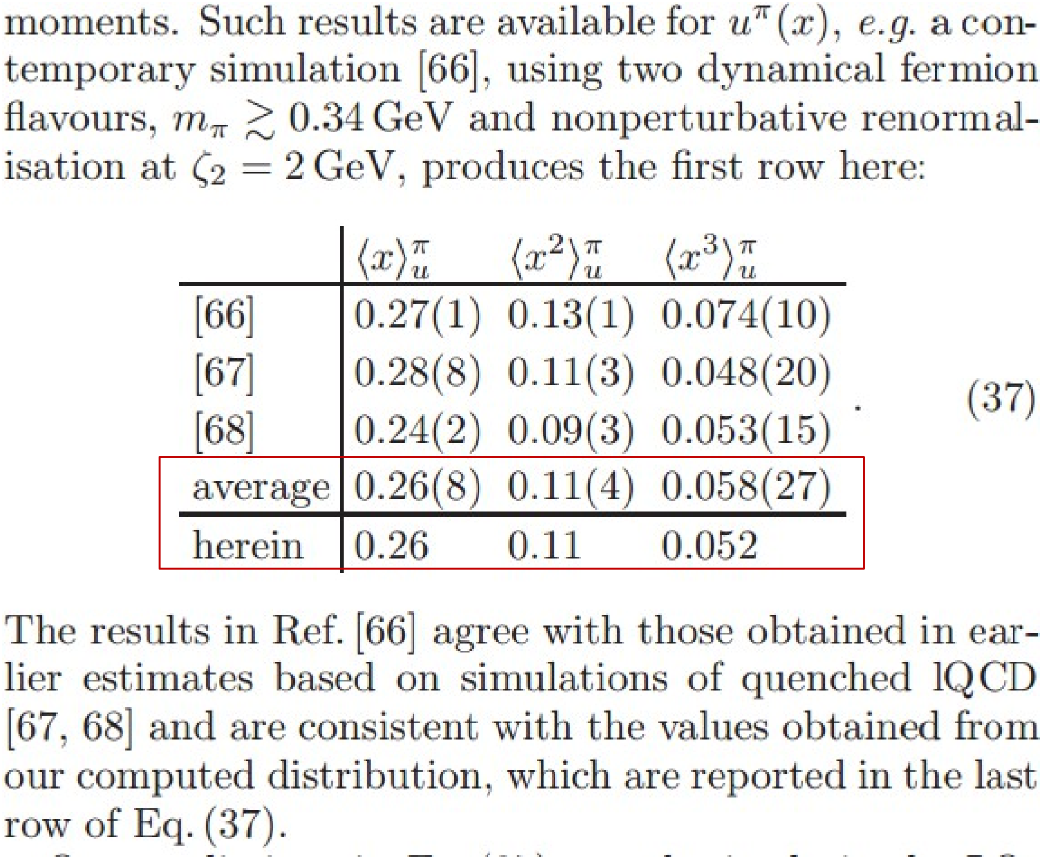


FIG. 1. Valence-quark PDFs at the hadronic scale, ζ_H, defined by Eqs. (14), (17): $u_V^K(x)$, solid (black) curve; $\bar{s}_V^K(x)$, dashed (blue) curve; $u_V^\pi(x)$, dot-dashed (green) curve; and $[u_V^K(x)+\bar{s}_V^K(x)]/2$ dotted (red) curve.

Pion:
DSE comparison with lQCD moments

- All lQCD studies agree with each other, within errors
- DSE and lQCD agree, within errors; and DSE at level of 4% with lQCD-average
- [66]: Brommel et al. (2007)
  [67]: Best *et al*. (1997)

  [68]: Detmold *et al*. (2003)
- On light-front, just 52% of the pion's momentum is carried by valence-quarks at *ζ2* = 2GeV, down from 65% at *ζH*= 0.51GeV



moments. Such results are available for $u^\pi(x)$, *e.g.* a contemporary simulation [66], using two dynamical fermion flavours, $m_\pi \gtrsim 0.34\,\text{GeV}$ and nonperturbative renormalisation at $\zeta_2 = 2\,\text{GeV}$, produces the first row here:

	$\langle x \rangle_u^\pi$	$\langle x^2 \rangle_u^\pi$	$\langle x^3 \rangle_u^\pi$
[66]	0.27(1)	0.13(1)	0.074(10)
[67]	0.28(8)	0.11(3)	0.048(20)
[68]	0.24(2)	0.09(3)	0.053(15)
average	0.26(8)	0.11(4)	0.058(27)
herein	0.26	0.11	0.052

. (37)

The results in Ref. [66] agree with those obtained in earlier estimates based on simulations of quenched lQCD [67, 68] and are consistent with the values obtained from our computed distribution, which are reported in the last row of Eq. (37).