Non-Peturbative Aspects of QCD below and above the Phase Transition

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References

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- **Hadron Resonance Gas for Polyakov loop:** Phys.Rev.Lett. 109 (2012) 151601. arXiv:1207.4875 [hep-ph]. arXiv:1207.7287 [hep-ph] Acta Phys. Polon. B **45**, 2407 (2014)
- **Polyakov loop Spectroscopy:** Phys.Rev. D89 (2014) 076006 109.

Outline

- The Hadron Spectrum
- Quarks and gluons at finite temperature
- Hadron Resonance Gas from Chiral Quark Models
- Conclusions

Motivation

- The partition function of QCD

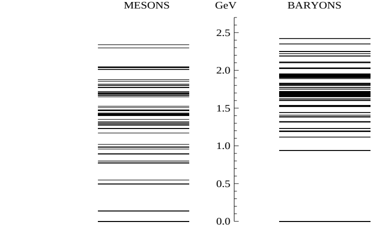
$$Z_{\rm QCD} = \sum_n e^{-E_n/T} \qquad H_{\rm QCD}\psi_n = E_n\psi_n$$

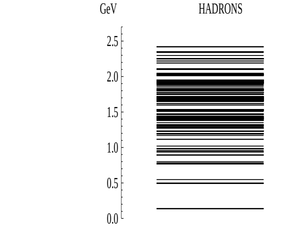
- Spectrum of QCD $\to$ Thermodynamics
- Colour singlet states (hadrons + ...???)
- Do we see quark-gluon substructure BELOW the “phase transition” ?

HADRONIC SPECTRUM AT ZERO TEMPERATURE

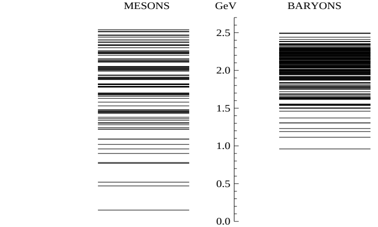
Hadron Spectrum (u,d,s)

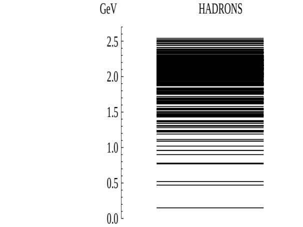
- Particle Data Group (PDG) compilation





- Relativized Quark Model (RQM)



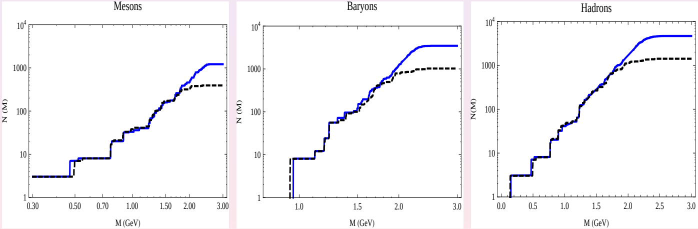


Cumulative number of states

- Compare $H_{\rm QCD}, H_{\rm PDG}, H_{\rm RQM}$ with staircase function

$$N(M) = \sum_n \theta(M - M_n)$$

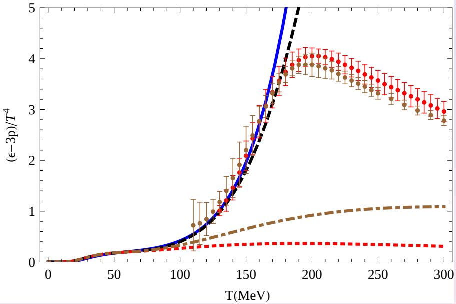
- Which states count ?
- Is $N_{\rm QCD}(M)$ accessible ?



$N_{q\bar q} \sim M^6$ $N_{qqq} \sim M^{12}$ $N_{\bar q q \bar q q} \sim M^{18}$ $N_{\rm hadrons} \sim e^{M/T_H}$

$T_H \sim 150{\rm MeV}$= Hagedorn temperature

QCD Spectrum and Trace anomaly



$$\mathcal{A}_{\rm HRG}(T) \equiv \frac{\epsilon - 3P}{T^4} = \frac{1}{T^4} \sum_n \int \frac{d^3p}{(2\pi)^3} \frac{E_n(p) - \vec{p} \cdot \vec{\nabla}_p E_n(p)}{e^{E_n(p)/T} + \eta_n} \,,$$

$$E_n(p) = \sqrt{p^2 + M_n^2} \qquad \eta_n = \pm 1$$

- Non-interacting Hadron-Resonance Gas works for $T < 0.8\, T_c$
- Spectrum $\rightarrow$ Thermodynamics

Quark-Hadron duality at zero temperature

- In the confined phase we expect all observables to be represented by hadronic degrees of freedom.
  1. Gell-Mann–Oakes–Renner relation

$$2 \underbrace{\langle \bar{q} q \rangle m_q}_{\text{quarks}} = - \underbrace{f_\pi^2 m_\pi^2}_{\text{hadrons}} , \qquad (1)$$

  2. Transition form factor of the pion
  3. Effective Chiral lagrangians with resonances
  4. Deep inelastic scattering
- Are hadrons a complete set of states ?
- Is the PDG complete or overcomplete ?
- The “phase transition” is a smooth cross-over, so we expect to see departures from quark-hadron duality below T_c

Static energies and Casimir scaling

The interaction between heavy sources A and B in perturbation theory

$$V_{AB}(r) = \lambda_A \cdot \lambda_B \frac{\alpha_S}{r}$$

The interaction between heavy sources on the lattice

$$E_{AB}(r) = \lambda_A \cdot \lambda_B \left[\frac{\alpha_S}{r} + \sigma r\right]$$

Casimir scaling requires that the ratio between the fundamental $Q\bar{Q} \equiv \mathbf{3} \times \bar{\mathbf{3}}$ and adjoint $GG \equiv \mathbf{8} \times \mathbf{8}$ colour sources are

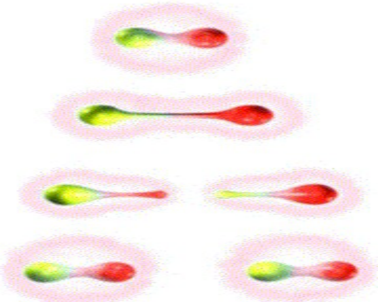
$$V_{Q\bar{Q}}(r) = \sigma_F r - \frac{4\alpha_s}{3r} + \dots \quad (2)$$

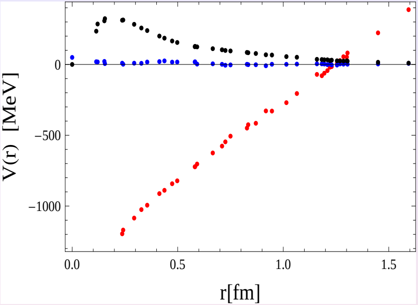
$$V_{GG}(r) = \sigma_A r - \frac{3\alpha_s}{r} + \dots \quad (3)$$

$$\frac{\sigma_A}{\sigma_F} = \frac{9}{4} \quad (4)$$

Quark potential and string breaking

Transition $Q\bar{Q} \to B\bar{B}$





- Energy of two heavy quarks

$$E(r) = m_{\bar{Q}} + m_Q + V(r)$$

- Meson masses

$$M_{\bar{q}Q} = \Delta_{\bar{q}Q} + m_Q \qquad M_{q\bar{Q}} = \Delta_{q\bar{Q}} + m_Q$$

- Uncoupled Born-Oppenheimer (diabatic crossings)

$$V_{\bar{Q}Q}(r) = \sigma r \,, \qquad V_{\bar{Q}q\bar{q}Q}(r) = \Delta_{\bar{q}Q} + \Delta_{\bar{Q}q} = 2\Delta \,.$$

Excited states

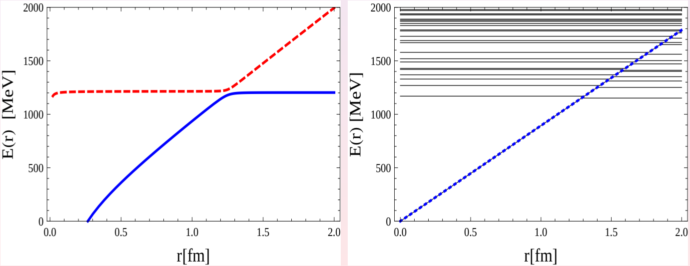
- Estimate of the string breaking distance $M_{q\bar{Q}} = 2M_0 + m_q + m_Q$

$$\sigma r_c = 2M_{q\bar{Q}} - 2m_Q \sim 4M_0$$

(constituent quark mass) $\rightarrow r_c \sim 1.2\mathrm{fm}$

- In general many excited meson states

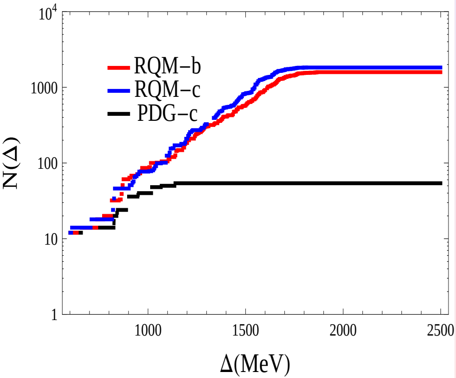
$$V^{(0,0)}_{\bar{Q}Q}(r) = \sigma r\,, \qquad V^{(n,m)}_{\bar{Q}q,\bar{q}Q}(r) = \Delta^{(n)}_{q\bar{Q}} + \Delta^{(m)}_{\bar{q}Q}\,,$$



Spectrum with one heavy quark

$$N(\Delta) = \sum_n \theta(\Delta - \Delta_n) \sim N_{\bar{q}Q}(\Delta) + N_{Qqq}(\Delta) + \cdots \sim e^{\Delta/T_H}$$

T_H Hagedorn temperature for hadrons with ONE heavy quark

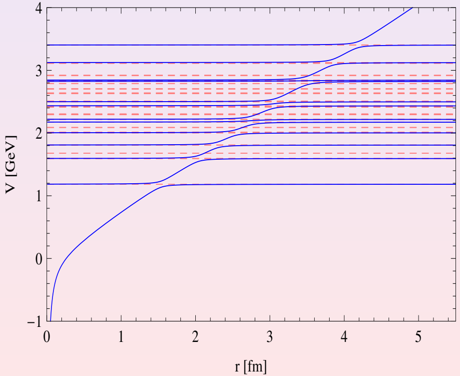


Avoided crossings

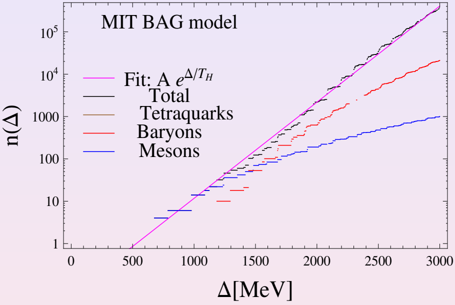
- Transition potential $V_{\bar{Q}\bar{Q}\to\bar{B}B}(r)$. Coupled channels

$$\begin{pmatrix} V_{\bar{Q}Q}(r) & V_{\bar{Q}\bar{Q}\to\bar{B}B}(r) \\ V_{\bar{Q}\bar{Q}\to\bar{B}B}(r) & V_{\bar{B}\bar{B}}(r) \end{pmatrix}$$

- Avoided crossing with states having the same quantum numbers as QQ

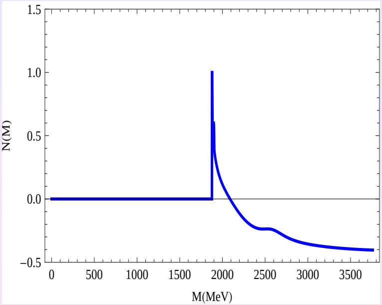


Hagedorn and The bootstrap



Which are the complete set of states in the PDG ?
Should X,Y,Z's or the deuteron or ^{208}Pb enter as multiquark states ?

Who counts ?



- The cumulative number in a given channel in the continuum with threshold $M_{\rm th}$

$$N(M) = \sum_n \theta(M - M_n) + [\delta(M) - \delta(M_{\rm th})]/\pi$$

- Levinson's theorem

$$N(\infty) = n_B + [\delta(\infty) - \delta(M_{\rm th})]/\pi = 0$$

- Deuteron doesn't count

QUARKS AND GLUONS AT FINITE TEMPERATURE

QCD at finite temperature

QCD Lagrangian:

$$\mathcal{L}_{\rm QCD} = -\frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu} + \sum_f \overline{q}^a_f (i\gamma_\mu D_\mu - m_f) q^a_f \, ;$$

Partition function

$$\begin{aligned} Z_{\rm QCD} &= {\rm Tr} e^{-H/T} = \sum_n e^{-E_n/T} \\ &= \int \mathcal{D} A_{\mu,a} \exp\left[-\frac{1}{4} \int d^4x (G^a_{\mu\nu})^2 \right] {\rm Det}(i\gamma_\mu D_\mu - m_f) \end{aligned}$$

Boundary conditions and Matsubara frequencies

$$q(\vec{x},\beta) = -q(\vec{x},0) \qquad A_\mu(\vec{x},\beta) = A_\mu(\vec{x},0) \qquad \beta = 1/T$$

$$\int \frac{dp_0}{2\pi} f(p_0) \to T \sum_n f(w_n)$$

$$w_n = (2n+1)\pi T \qquad w_n = 2n\pi T$$

Thermodynamic relations

- Statistical mechanics of non-interacting particles

$$\log Z = V\eta g_i \int \frac{d^3p}{(2\pi)^3} \log\left[1+\eta e^{-E_p/T}\right] \qquad E_p = \sqrt{p^2+m^2}$$

$\eta = -1$ for bosons ; $\eta = -1$ for fermions ; g_i-number of species

$$\begin{aligned} F &= -T\log Z \qquad P = -T\frac{\partial F}{\partial V} \\ S &= -\frac{\partial(TF)}{\partial T} \qquad E = F + TS \end{aligned}$$

- High temperature limit $\rightarrow$ Free gas of gluons and quarks

$$P = \left[2(N_c^2-1) + 4N_cN_f\frac{7}{8}\right]\frac{\pi^2}{90}T^4$$

Interaction measure (trace anomaly)

$$\Delta \equiv \frac{\epsilon - 3p}{T^4} \rightarrow 0 \qquad (T\rightarrow\infty)$$

Symmetries in QCD

Colour gauge invariance

$$q(x) \to e^{i\sum_a (\lambda_a)^c \alpha_a(x)} q(x) \equiv g(x) q(x)$$
$$A_\mu^g(x) = g^{-1}(x)\partial_\mu g(x) + g^{-1}(x) A_\mu(x) g(x)$$

Only periodic gauge transformations are allowed:

$$g(\vec{x}, x_0 + \beta) = g(\vec{x}, x_0)\,, \qquad \beta = 1/T\,.$$

In the static gauge $\partial_0 A_0 = 0$

$$g(x_0) = e^{i2\pi x_0 \lambda/\beta}\,, \quad \text{where} \quad \lambda = \mathrm{diag}(n_1, \cdots, n_{N_c})\,, \quad \mathrm{Tr}\lambda = 0\,.$$

Large Gauge Invariance: $\Rightarrow$ periodicity in A_0 with period $2\pi/\beta$

$$A_0 \to A_0 + 2\pi T \mathrm{diag}(n_j) \qquad \text{Gribov copies}$$

Explicitly Broken in perturbation theory (non-perturbative finite temperature gluons)

Symmetries in QCD

In the limit of massless quarks ($m_f = 0$),

- Invariant under scale

$$(\mathbf{x} \longrightarrow \lambda \mathbf{x})$$

Broken by quantum corrections regularization (Trace anomaly)

$$\epsilon - 3p = \frac{\beta(g)}{2g} \langle (G^a_{\mu\nu})^2 \rangle \neq 0 \,,$$

- Chiral **Left** $\leftrightarrow$ **Right** transformations.

$$q(x) \to e^{i \sum_a (\lambda_a)^f \alpha_a} q(x) \qquad q(x) \to e^{i \sum_a (\lambda_a)^f \alpha_a \gamma_5} q(x)$$

Broken by chiral condensate in the vacuum

$$\langle \bar{q} q \rangle \neq 0$$

Symmetries in QCD

Gluodynamics: In the limit of heavy quarks ($m_f \to \infty$)

$$Z \to \int \mathcal{D}A_{\mu,a} \exp\left[-\frac{1}{4}\int d^4x (G^a_{\mu\nu})^2\right] \mathrm{Det}(-m_f)$$

Larger symmetry ('t Hooft) Center Symmetry $\mathbb{Z}(N_c)$

$$g(\vec{x}, x_0+\beta) = z\, g(\vec{x}, x_0)\,, \qquad z^{N_c} = 1, \quad (z \in \mathbb{Z}(N_c))\,.$$

$$g(x_0) = e^{i2\pi x_0 \lambda/(N_c\beta)}\,, \qquad A_0 \to A_0 + \frac{2\pi T}{N_c}\mathrm{diag}(n_j)$$

The Poyakov loop

$$L_T = \frac{1}{N_c}\langle \mathrm{tr}_c e^{iA_0/T}\rangle = e^{-F_q/T} = e^{i2\pi/N_c} L_T = 0$$

$F_q = \infty$ means CONFINEMENT
At high temperatures $A_0/T << 1$

$$L_T = 1 - \frac{\langle \mathrm{tr}_c A_0^2\rangle}{2N_c T^2} + \cdots = e^{-\frac{\langle \mathrm{tr}_c A_0^2\rangle}{2N_c T^2}+\ldots}$$

In full QCD $L_T = \mathcal{O}(e^{-m_q/T}) \neq 0$ Large Violation of center sym.

Lattice results in full QCD

The chiral-deconfinement cross over is a unique prediction of lattice QCD

- Order parameter of chiral symmetry breaking ($m_q = 0$)
  Quark condensate $SU(N_f) \otimes SU(N_f) \rightarrow SU_V(N_f)$

$$\langle \bar{q}q \rangle \neq 0 \quad T < T_c \qquad \langle \bar{q}q \rangle = 0 \quad T > T_c$$

- Order parameter of deconfinement ($m_q = \infty$)
  Polyakov loop: Center symmetry $Z(N_c)$ broken

$$L_T = \frac{1}{N_c} \langle \mathrm{tr}_c e^{iA_0/T} \rangle = 0 \quad T < T_c \qquad L_T = \frac{1}{N_c} \langle \mathrm{tr}_c e^{iA_0/T} \rangle = 1 \quad T > T_c$$

- In the real world m_q is finite. The chiral-deconfinement crossover (connected) crossed correlator (never computed on lattice),

$$\langle \bar{q}q \, \mathrm{tr}_c \, e^{igA_0/T} \rangle - \langle \bar{q}q \rangle \langle \mathrm{tr}_c \, e^{igA_0/T} \rangle = \frac{\partial L_T}{\partial m_q} , \tag{5}$$

Polyakov loop correlators in higher representations

Polyakov line in the fundamental representation

$$\Omega_F(\vec{x}) = e^{iA_0(\vec{x})/T} \qquad A_0 = \sum_{a=1}^{N_c^2-1} \lambda_a A_0^a$$

The interaction between heavy sources A and B (Yang-Mills)

$$\langle \mathrm{Tr}_F \Omega(\vec{x}_1) \mathrm{Tr}_F \Omega(\vec{x}_2)^\dagger \rangle \to e^{-\sigma_F |\vec{x}_1 - \vec{x}_2|/T}$$

In general (Casimir scaling)

$$\langle \mathrm{Tr}_R \Omega(\vec{x}_1) \mathrm{Tr}_R \Omega(\vec{x}_2)^\dagger \rangle \to e^{-\sigma_R |\vec{x}_1 - \vec{x}_2|/T} \qquad \sigma_R = (C_R/C_F)\sigma_F$$

Quark free energy in QCD

- Spectral decomposition with integral weights w_n and positive energies $E_n(|\vec{x}_1 - \vec{x}_2|) > 0$,

$$\langle \mathrm{Tr}_F \Omega(\vec{x}_1) \mathrm{Tr}_F \Omega(\vec{x}_2)^\dagger \rangle = \sum_n w_n e^{-E_n(|\vec{x}_1 - \vec{x}_2|)/T} = e^{-F(r,T)/T} ,$$

At large distances

$$\langle \mathrm{Tr}_F \Omega(\vec{r}) \mathrm{Tr}_F \Omega(0)^\dagger \rangle \to |\langle \mathrm{Tr}_F \Omega \rangle|^2 \equiv L_T^2 . \qquad (6)$$

Neglect avoided crossings $\to w_0 = 1$ and $E_0(r) = V_{\bar{Q}Q}(r) = \sigma r - \pi/12r$,

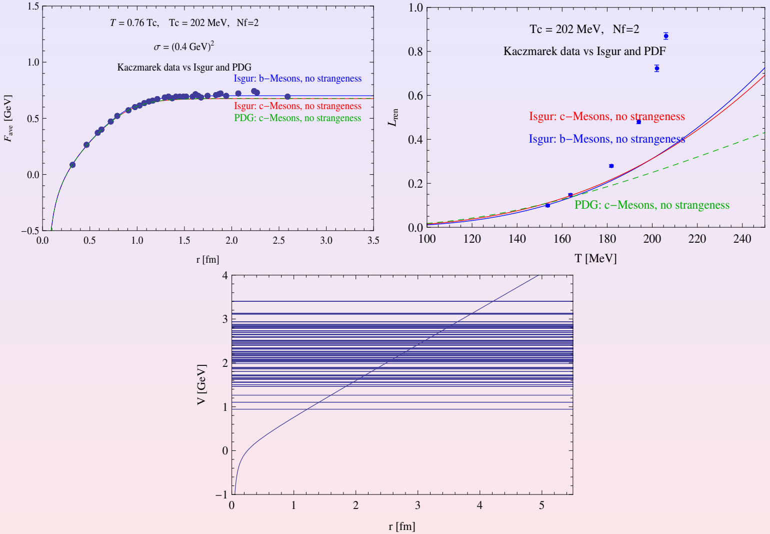
$$\begin{aligned} e^{-F(r,T)/T} &= \langle \mathrm{Tr}_F \Omega(\vec{r}) \mathrm{Tr}_F \Omega(0)^\dagger \rangle = \sum_{n,m} e^{-V_{\bar{Q}Q}^{(n,m)}(r)/T} \\ &= e^{-V_{\bar{Q}Q}(r)/T} + L_T^2 \end{aligned}$$

$\Delta_n = \Delta^{(n)}_{\bar{q}Q} = \Delta^{(n)}_{q\bar{Q}}$ (charge conjugation)

- Polyakov loop $M_0 \sim 350\mathrm{MeV}$ (chiral symmetry breaking)

$$L_T = \sum_n e^{-\Delta_n/T} \sim e^{-2M_0/T} \ll 1 ,$$

Quark free energy



$$F(r,T) = -T \log \left[e^{-V_{\bar{Q}Q}(r)/T} + e^{-F(\infty,T)/T} \right] , \qquad (7)$$

INSIGHTS FROM GLUODYNAMICS

Symmetries in QCD

Gluodynamics: In the limit of heavy quarks ($m_f \to \infty$)

$$Z \to \int \mathcal{D}A_{\mu,a} \exp\left[-\frac{1}{4}\int d^4x (G^a_{\mu\nu})^2\right] \mathrm{Det}(-m_f)$$

Larger symmetry ('t Hooft) Center Symmetry $\mathbb{Z}(N_c)$

$$g(\vec{x}, x_0 + \beta) = z\, g(\vec{x}, x_0)\,, \qquad z^{N_c} = 1, \quad (z \in \mathbb{Z}(N_c))\,.$$

$$g(x_0) = e^{i2\pi x_0 \lambda/(N_c \beta)}\,, \qquad A_0 \to A_0 + \frac{2\pi T}{N_c}\mathrm{diag}(n_j)$$

The Poyakov loop

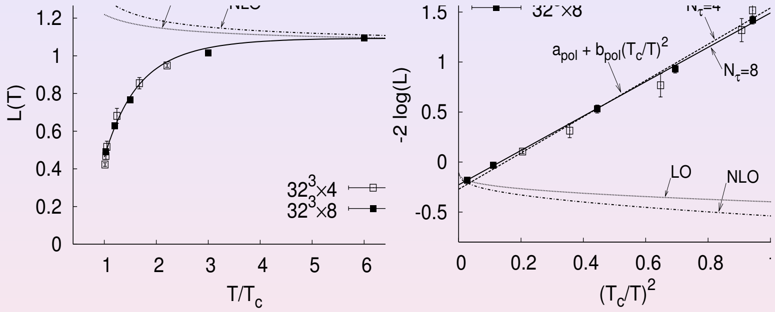
$$L_T = \frac{1}{N_c}\langle \mathrm{tr}_c e^{iA_0/T}\rangle = e^{-F_q/T} = e^{i2\pi/N_c} L_T = 0$$

$F_q = \infty$ means CONFINEMENT
At high temperatures $A_0/T << 1$

$$L_T = 1 - \frac{\langle \mathrm{tr}_c A_0^2\rangle}{2N_c T^2} + \cdots = e^{-\frac{\langle \mathrm{tr}_c A_0^2\rangle}{2N_c T^2} + \dots}$$

In full QCD $L_T = \mathcal{O}(e^{-m_q/T}) \neq 0 \ll 1$

Power temperature corrections in the Polyakov loop



$$-2\log(L) = a_{\rm P} + \frac{a_{\rm NP}}{T^2}\,, \quad a_{\rm NP} = (1.81 \pm 0.13)\,T_c^2\,, \qquad 1.03\,T_c < T < 6\,T_c\,.$$

Perturbative result fails to reproduce lattice data in this regime.

Trace Anomaly

Partition function (gluodynamics $m_f \to \infty$) $\bar{A}_\mu = gA_\mu$

$$Z = \int \mathcal{D}\bar{A}_{\mu,a} \exp\left[-\frac{1}{4g^2}\int d^4x (\bar{G}^a_{\mu\nu})^2\right]$$

$$\frac{\partial \log Z}{\partial g} = \frac{1}{2g^3}\left\langle \int d^4x (\bar{G}^a_{\mu\nu})^2 \right\rangle = \frac{1}{2g}\frac{V}{T}\langle (G^a_{\mu\nu})^2 \rangle$$

Free energy and Total Energy

$$F = -PV = -T\log Z \qquad \epsilon = \frac{E}{V} = \frac{T^2}{V}\frac{\partial \log Z}{\partial T} \tag{8}$$

$$\epsilon - 3P = T^5 \frac{\partial}{\partial T}\left(\frac{P}{T^4}\right) . \tag{9}$$

Renormalization scale μ

$$\frac{P}{T^4} = f(g(\mu), \log(\mu/2\pi T)). \tag{10}$$

$$\frac{\partial}{\partial \log T}\left(\frac{P}{T^4}\right) = \frac{\partial g}{\partial \log \mu}\frac{\partial}{\partial g}\left(\frac{P}{T^4}\right) \tag{11}$$

The trace anomaly

$$\epsilon - 3P = \frac{\beta(g)}{2g}\langle (G^a_{\mu\nu})^2\rangle ,$$

where we have introduced the beta function

$$\beta(g) = \mu\frac{dg}{d\mu} = -\frac{11N_c}{48\pi^2}g^3 + \mathcal{O}(g^5) . \tag{12}$$

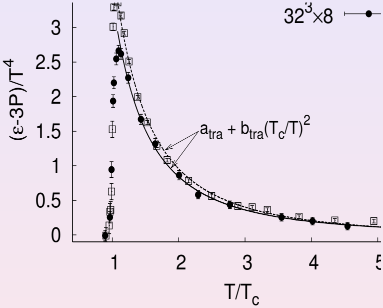
Perturbation theory to two loops (J.I.Kapusta, NPB148 (1979)):

$$\Delta \equiv \frac{\epsilon - 3p}{T^4} = \frac{N_c(N_c^2-1)}{1152\pi^2}\beta_0 g(T)^4 + \mathcal{O}(g^5)$$

where $1/g^2(\mu) = \beta_0 \log(\mu^2/\Lambda^2_{\rm QCD})$

Power temperature corrections from Lattice data

Trace Anomaly $N_c = 3, N_f = 0$
G. Boyd et al., Nucl. Phys. B469, 419 (1996).



$$\frac{\epsilon - 3P}{T^4} = a_{\rm P} + \frac{a_{\rm NP}}{T^2}\,, \quad a_{\rm NP} = (3.46 \pm 0.13)\, T_c^2\,, \qquad 1.13\, T_c < T < 4.5\, T_c\,.$$

The fuzzy bag of Pisarski

Low temperature (confined) $\rightarrow$ glueball gas

$$P_{\text{glueball}}(T) =\sim e^{-M_G/T} \qquad M_G \gg T_c \rightarrow P_{\text{glueball}}(T_c) = 0$$

High temperature (deconfined) $\rightarrow$ free gluon gas

$$P_{\text{gluons}}(T) = \frac{b_0}{2} T^4 \qquad b_0 = \frac{(N_c^2 - 1)\pi^2}{45}$$

Pisarski's (temperature dependent) fuzzy bag , PTP 2006

$$P(T) = P_{\text{gluons}}(T) - B_{\text{fuzzy}}(T), \qquad T > T_c\,, \qquad P(T_c) = P_{\text{glueballs}}(T_c) = 0$$

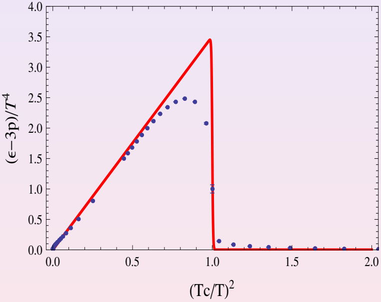
$$B_{\text{fuzzy}} = \frac{b_0}{2} T_c^2 T^2 \qquad \rightarrow \qquad P = \frac{b_0}{2}(T^4 - T^2 T_c^2)$$

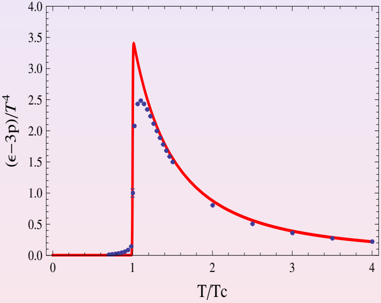
Then

$$\Delta \equiv \frac{\epsilon - 3P}{T^4} = b_0 \left(\frac{T_c}{T}\right)^2 \qquad b_0 = 3.45(3.5 Fit!!!!) \tag{13}$$

Power temperature corrections from Lattice data

Trace Anomaly $N_c = 3, N_f = 0$
JHEP Wuppertal 2012





$$\Delta(T) = \frac{(N_c^2 - 1)\pi^2}{45} \left(\frac{T_c}{T}\right)^2 \theta(T - T_c)$$

Static energies and Casimir scaling

The interaction between heavy sources A and B in perturbation theory

$$V_{AB}(r) = \lambda_A \cdot \lambda_B \frac{\alpha_S}{r}$$

The interaction between heavy sources on the lattice

$$E_{AB}(r) = \lambda_A \cdot \lambda_B \left[\frac{\alpha_S}{r} + \sigma r \right]$$

Casimir scaling requires that the ratio between the fundamental $Q\bar{Q} \equiv \mathbf{3} \times \bar{\mathbf{3}}$ and adjoint $GG \equiv \mathbf{8} \times \mathbf{8}$ colour sources are

$$V_{Q\bar{Q}}(r) = \sigma_F r - \frac{4\alpha_s}{3r} + \dots \quad (14)$$

$$V_{GG}(r) = \sigma_A r - \frac{3\alpha_s}{r} + \dots \quad (15)$$

$$\frac{\sigma_A}{\sigma_F} = \frac{9}{4} \quad (16)$$

Polyakov loop correlators in higher representations

Polyakov line in the fundamental representation

$$\Omega_F(\vec{x}) = e^{iA_0(\vec{x})/T} \qquad A_0 = \sum_{a=1}^{N_c^2-1} \lambda_a A_0^a$$

The interaction between heavy sources A and B

$$\langle \mathrm{Tr}_F \Omega(\vec{x}_1) \mathrm{Tr}_F \Omega(\vec{x}_2)^\dagger \rangle \to e^{-\sigma_F |\vec{x}_1 - \vec{x}_2|/T}$$

In general (Casimir scaling)

$$\langle \mathrm{Tr}_R \Omega(\vec{x}_1) \mathrm{Tr}_R \Omega(\vec{x}_2)^\dagger \rangle \to e^{-\sigma_R |\vec{x}_1 - \vec{x}_2|/T} \qquad \sigma_R = (C_R/C_F)\sigma_F$$

Glueball spectrum

Two masless spin-1 particles in CM system. Salpeter equation for the mass operator

$$\hat{M} = 2p + \sigma_A r \qquad \sigma_A = \frac{9}{4}\sigma$$

Uncertainty principle for the ground state $pr \sim 1$

$$M_0 = \min \left[\frac{2}{r} + \sigma_A r \right] = 2\sqrt{2\sigma_A} = 3.4\sqrt{\sigma}$$

WKB spectrum for excited states. Bohr-Sommerfeld quantization condition (L=0)

$$2 \int_0^a dr p_r = 2(n+\alpha)\pi \quad \rightarrow M_n^2 = 4\pi\sigma_A (n+\alpha)$$

Glueball spectrum of two gluons

$$(2p + \sigma_A r)\psi_n = M_n \psi_n$$

Harmonic oscillator wave functions

$$R_{nl}(r) = \frac{u_{nl}(r)}{r} = \frac{1}{\sqrt[4]{\pi}} e^{-\frac{r^2}{2b^2}} \left(\frac{r}{b}\right)^l \sqrt{\frac{(n-1)!2^{l+n+1}}{b^3(2l+2(n-1)+1)!!}} L_{n-1}^{l+\frac{1}{2}}\left(\frac{r^2}{b^2}\right)$$

$L_{n-1}^{l+\frac{1}{2}}(x)$ are asociated Laguerre polynomials.

$$-u''_{nl}(r) + \left[\frac{r^2}{b^4} + \frac{l(l+1)}{r^2}\right] u_{nl}(r) = \frac{1}{b^2}(2l+4n-1)u_{nl}(r)$$

Normalization

$$\int_0^\infty dr r^2 R_{nl}(r)^2 = \int_0^\infty dr u_{nl}(r)^2 = 1$$

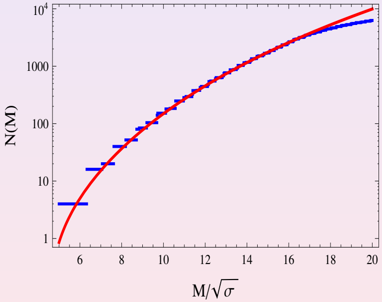
where b has dimensions of length. The single-particle energies are

$$\epsilon_{nl} = \frac{1}{2Mb^2}(4n+2l-1) = \omega\,(2n+l-1/2)$$

where the oscillator frequency is $\omega = 1/(Mb^2)$.

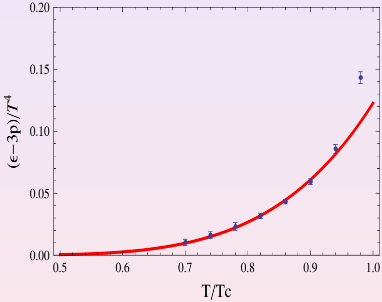
At large masses a derivative expansion at long distances

$$N_{2g}(M) \rightarrow g^2 \int \frac{d^3x d^3p}{(2\pi)^3} \theta(M - H(p,r)) + \mathcal{O}(\nabla H)$$
$$= \frac{g^2 M^6}{720\pi\sigma_A^3} + \frac{\alpha_s g^2 M^4}{16\pi\sigma_A^2} + \frac{9\alpha_s^2 g^2 M^2}{8\pi\sigma_A} - \frac{g^2 M^2}{9\pi\sigma_A} + \dots$$



Trace anomaly

$$\Delta^{2g}_{\text{glueball}}(T) = \sum_n \frac{1}{2\pi^2 k} K_1\left(\frac{kM_n}{T}\right)\left(\frac{M_n}{T}\right)^3$$



$$\frac{T_c}{\sqrt{\sigma}} = 0.736385 \qquad \text{Lattice} \quad 0.629(3)$$

Trace anomaly (WKB)

$$\Delta(T) = \sum_{k=1}^{\infty} \int dM \frac{\partial N(M)}{\partial M} \frac{1}{2k\pi^2} \left(\frac{M}{T}\right)^3 K_1\left(k\frac{M}{T}\right)$$

Large M expansion $\rightarrow$ Large T expansion

$$N(M) = \sum_n a_n M^n$$

$$\int_0^\infty nM^{n-1} \left(\frac{M}{T}\right)^3 \frac{1}{2\pi^2 k} K_1(kM/T) = \frac{2^n n k^{-n-4} T^n \Gamma\left(\frac{n}{2}+1\right)\Gamma\left(\frac{n}{2}+2\right)}{\pi^2}$$

$$\Delta(T) = \sum_n a_n \frac{2^n n T^n \zeta(n+4)\Gamma\left(\frac{n}{2}+1\right)\Gamma\left(\frac{n}{2}+2\right)}{\pi^2}$$

$$\Delta_{2g}(T) = \frac{2048\pi^8}{3465} a_6 T^6 + \frac{128\pi^6}{1575} a_4 T^4 + \frac{128\pi^6}{1575} a_2 T^2$$

Multigluon states

$$H_n = \sum_{i=1}^{N} p_i + \sum_{i<j} \sigma_A |\vec{x}_i - \vec{x}_j| \tag{17}$$

In the CM system

$$N_n(M) \sim \int \prod_{i=1}^{N} \frac{d^3x_i d^3p_i}{(2\pi)^3} \theta(M - H_n)\delta(\sum_i \vec{x}_i)\delta(\sum_i \vec{p}_i) \sim \left(\frac{M^2}{\sigma_A}\right)^{6n-6} \tag{18}$$

$$\Delta_{ng}(T) \sim \left(\frac{T^2}{\sigma_A}\right)^{6n-6} \tag{19}$$

Scale separation between 2g-WKB and 3g glueballs

$$\Delta_{3g}(T) \sim e^{-M_{3g}/T} << \Delta_{2g}(T) \tag{20}$$

Gluelump spectrum

One masless spin-1 particle and one gluon source (infinitely heavy) in CM system. Salpeter equation for the mass operator

$$\hat{\Delta} = p + \sigma_A r \qquad \rightarrow \quad M_{\rm gluelump} = M_{\rm glueball}/\sqrt{2}$$

The smallest mass gap is the gluelump not the glueball !
The partition function

$$Z_{\rm gluelumps}(T) = Z_{\rm glueballs}(T/\sqrt{2})/g$$

Quark-Hadron duality for the Polyakov loop at low temperatures

$$\langle \Omega_8 \rangle_T \sim Z_{\rm gluelumps}(T) = \sum_n e^{-\Delta_n/T} \neq 0 \quad (T < T_c)$$

Higher representations in the gauge group, multigluon states ...

FROM CHIRAL QUARK MODELS TO THE HADRON RESONANCE GAS

Minimal coupling of the Polyakov loop to Chiral Quark models

Constituent Quark model:

$$\mathcal{L}_{\rm QC} = \overline{q}\,\mathbf{D}\,q\,, \qquad \mathbf{D} = \partial\!\!\!/ + V\!\!\!/^{f} + A\!\!\!/^{f} + MU^{\gamma_5} + \hat{m}_0$$

Consider the minimal coupling of the gluons in the model:

$$V_\mu^f \longrightarrow V_\mu^f + gV_\mu^c\,, \quad V_\mu^c = \delta_{\mu 0} V_0^c$$

- We introduce a colour source (Polyakov loop).
- We obtain the (Peirls-Yoccoz) projection onto the color neutral states by integrating over the A_0 field.
- In Quenched approximation: Group integration in SU(N_c).

$$Z = \int DUD\Omega\, e^{-\Gamma_G[\Omega]}\, e^{-\Gamma_Q[U,\Omega]}$$

Quark-Hadron Duality at Finite Temperature (Polyakov loop)

Partition function for N_f-flavours

$$Z_{\rm HRG}(N_f) \equiv \int D\Omega \, e^{-S(N_f)} \qquad S(N_f) = S_q(N_f) + S_G$$

Quark contribution

$$S_q(N_f) = -2N_f \int \frac{d^3x d^3p}{(2\pi)^3} \left({\rm tr}_c \log \left[1 + \Omega(x) \, e^{-E_p/T} \right] + {\rm c.c.} \right)$$

One extra HEAVY QUARK (not anti-quark) with flavour a

$$S_q(N_f+1) - S_q(N_f) \quad = \quad -2 \log(1 + \Omega_{aa} e^{-E_h/T}) \approx -2 e^{-m_H/T} \Omega_{aa}$$

$$\frac{1}{N_c} \langle {\rm tr}_c \Omega \rangle = \lim_{m_H \to \infty} \frac{1}{2} \left[\frac{Z_{\rm HRG}(N_f+1)}{Z_{\rm HRG}(N_f)} - 1 \right] e^{m_H/T} = \frac{1}{2N_c} \sum_\alpha g_\alpha e^{-\Delta_\alpha/T}$$

$$\Delta_\alpha = \lim_{m_H \to \infty} (M_{H,\alpha} - m_H)$$

Quark-Hadron Duality at Finite Temperature (Quark free energy)

Four point correlator (cluster decomposition)

$$\langle \mathrm{tr}_c \Omega(\vec{x})^\dagger \mathrm{tr}_c \Omega(0) \mathrm{tr}_c \Omega(\vec{x}_1)^\dagger \mathrm{tr}_c \Omega(\vec{x}_2) \rangle_G = e^{-\sigma r/T} e^{-\sigma r_{12}/T} + e^{-\sigma |\vec{x}-\vec{x}_2|/T} e^{-\sigma r_1/T} \quad (21)$$

This yields

$$\langle \mathrm{tr}_c \Omega(\vec{x}) \mathrm{tr}_c \Omega^\dagger(0) \rangle = \frac{e^{-\sigma r/T} [1 + Z_{\bar{q}q} + \dots] + |\langle \mathrm{tr}_c \Omega \rangle|^2}{1 + Z_{\bar{q}q} + \dots} = e^{-\sigma r/T} + |\langle \mathrm{tr}_c \Omega \rangle|^2 \,. \quad (22)$$

This is the SAME as

$$F(r,T) = -T \log \left[e^{-V_{\bar{Q}Q}(r)/T} + e^{-F(\infty,T)/T} \right] , \quad (23)$$

Quantization of multiquark states

- Quantum and local Polyakov loop (PNJL models)
- Multiquark states: Create/Anhiquilate a quark at point $\vec{x}$ and momentum p

$$\Omega(x)e^{-E_P/T} \qquad \Omega(x)^{+}e^{-E_P/T}$$

- At low temperatures quark Boltzmann factor small $e^{-E_p/T} < 1$. The action becomes small

$$S_q[\Omega] = 2N_f \int \frac{d^3x d^3p}{(2\pi)^3} \left[\mathrm{tr}_c\Omega(x) + \mathrm{tr}_c\Omega(x)\right] e^{-E_p/T} + \dots$$

$$Z = \int D\Omega \, e^{-S[\Omega]} = \int D\Omega \left(1 - S[\Omega] + \frac{1}{2}S[\Omega]^2 + \dots\right)$$

- $\bar{q}q$ contribution

$$Z_{\bar{q}q} = (2N_f)^2 \int \frac{d^3x_1 d^3p_1}{(2\pi)^3} \int \frac{d^3x_2 d^3p_2}{(2\pi)^3} e^{-E_1/T} e^{-E_2/T} \underbrace{\langle \mathrm{tr}_c \Omega(\vec{x}_1) \mathrm{tr}_c \Omega^\dagger(\vec{x}_2) \rangle}_{e^{-\sigma|\vec{x}_1 - \vec{x}_2|/T}}$$

$$= (2N_f)^2 \int \frac{d^3x_1 d^3p_1}{(2\pi)^3} \frac{d^3x_2 d^3p_2}{(2\pi)^3} e^{-H(x_1,p_1;x_2,p_2)/T}$$

$\bar{q}q$ Hamiltonian

$$H(x_1, p_1; x_2, p_2) = E_1 + E_2 + V_{12}\,.$$

- Quantization in the CM frame $p_1 = -p_2 \equiv p$

$$\left(2\sqrt{p^2 + M^2} + V_{q\bar{q}}(r)\right) \psi_n = M_n \psi_n\,.$$

- Boosting the CM to any frame with momentum P

$$Z_{\bar{q}q} \to \sum_n \int \frac{d^3R d^3P}{(2\pi)^3} e^{-E_n(P)/T}$$

A GAS OF NON INTERACTING MESONS ! (valid to $\bar{q}q\bar{q}q$)

Polyakov loop in the quark model

$$L_T = 2N_f \int \frac{d^3x\, d^3p}{(2\pi)^3} e^{-E_p/T} \frac{1}{N_c} \underbrace{\langle \mathrm{tr}_c \Omega(\vec{x}_0) \mathrm{tr}_c \Omega^\dagger(\vec{x}) \rangle}_{e^{-\sigma[\vec{x}_0 - \vec{x}|/T}} + \cdots$$
$$= \frac{2N_f}{N_c} \int \frac{d^3x\, d^3p}{(2\pi)^3} e^{-H(\vec{x},\vec{p})/T}$$

Heavy-light ground state system

$$(\sqrt{p^2 + m_q^2} + \sigma r)\psi_n = \Delta_n \psi_n$$

In the limit $m_q \to 0$ we make $p \sim 1/r$ and $\Delta \sim 2\sqrt{\sigma} \sim 900\mathrm{MeV}$

$$N_c L(T) \sim 2N_f e^{-\Delta_M/T} + (2N_f^2 + N_f) e^{-\Delta_B/T} + \cdots = 21 e^{-\bar{\Delta}/T} \qquad (N_f = 3)$$

ENTROPY SHIFTS

Thermodynamic shifts

- Add one extra heavy charge belonging to rep R to the vacuum
- Energy of the states changes under the presence of the charge

$$E_n \to E_n^R \to \Delta_n^R + m_R + \dots$$

- In the static gauge $\partial_0 A_0 = 0$ the Ployakov loop operator

$$tr\Omega(\vec{r}) = \mathrm{tr} e^{iA_0(\vec{r})/T}$$

- The ratio of partition functions $\to$ Free energy shift

$$\langle tre^{iA_0/T} \rangle = \frac{Z_R}{Z_0} = e^{-\Delta F_R/T} = \frac{\sum_n e^{-\Delta_n/T}}{1 + \dots}$$

Counting states (One Hagedorn-Polyakov temperature)

$$N(\Delta) = \sum_n \theta(\Delta - \Delta_n) \sim e^{\Delta/T_{H,L}}$$

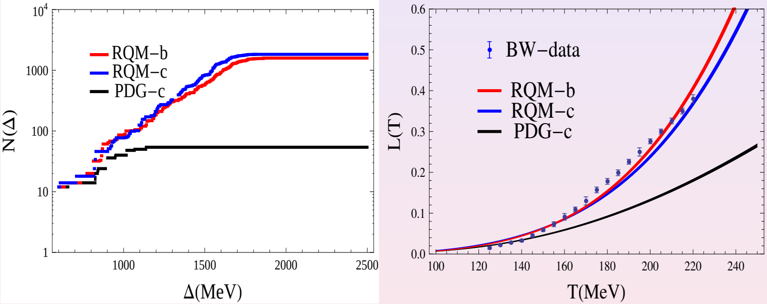


Figure: Left: $N(\Delta)$ as a function of the c-quark and b-quark mass subtracted hadron mass $\Delta = M - m_Q$ (in) with u, d and s quarks, computed in the RQM vs PDG. Right: Polyakov loop as a function of temperature (in MeV).

- Polyakov loop ambiguity removed by entropy shift

$$\langle \mathrm{tr}_R \Omega(0) \rangle_T = e^{-\Delta F_R(T)/T} \to \Delta S_R(T) = -\partial_T F_R(T)$$

- Third principle for degenerate states

$$\Delta S_Q(0) = \log(2N_f) \,, \qquad \Delta S_Q(\infty) = \log N_c$$

- RGE equation for specific heat

$$\Delta c_Q = T \frac{\partial S_Q}{\partial T} = \frac{\partial}{\partial T} \left\{ T \int d^4x \left[\frac{\langle \mathrm{tr}\Omega \, \Theta(x) \rangle}{\langle \mathrm{tr}\Omega \rangle} - \langle \Theta(x) \rangle \right] \right\} \equiv \frac{\partial U_Q}{\partial T}$$

- Energy momentum tensor

$$0 = \mu \frac{dS_Q}{d\mu} = \beta(g) \frac{\partial S_Q}{\partial g} - \sum_q m_q (1 + \gamma_q) \frac{\partial S_Q}{\partial m_q} - T \frac{\partial S_Q}{\partial T}$$

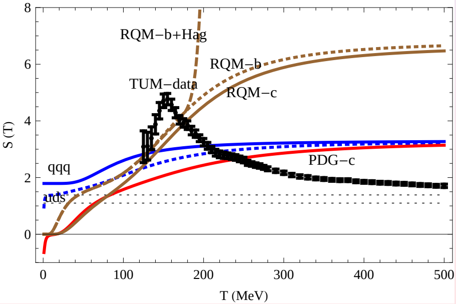
- Entropy shift IS NOT a true entropy $c = T \partial_T S = (\Delta H)^2 / T^2 > 0$

From Hadron resonance gas ...

- TUM collaboration $125 < T6000$MeV
- Constituent Quark Model $M = 300\mathrm{MeV}$, $m_u = 2.5\mathrm{MeV}, m_d = 5\mathrm{MeV}$, $m_s = 95\mathrm{MeV}$.

$$L = \sum_{q=u,d,s} g_q e^{-M_{\bar{Q}q}/T} + \sum_{q,q'=u,d,s} g_{q,q'} e^{-M_{\bar{Q}qq'}/T} + \dots$$

- All=($Q\bar{q}$, Qqq and $Q\bar{q}g$) Hadron spectrum (missing states !!)

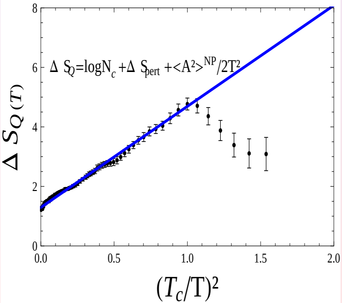


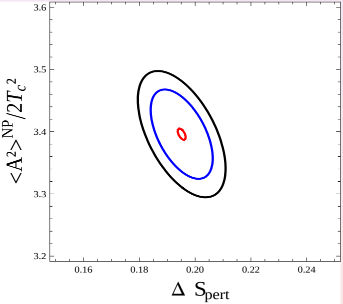
... to Power corrections

- Dim-2 condensates (AGAIN !)

$$\langle tr(e^{iA_o T})\rangle \sim N_c \exp\left[-g^2 \frac{\langle (A_0^a)^2\rangle}{4N_c T^2}\right] \to$$

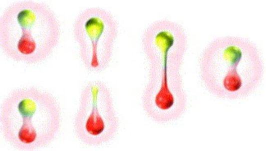
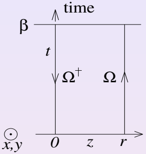
$$S_Q(T) = \frac{\langle \mathrm{tr}(\bar{A}_0^2)\rangle^{\mathrm{NP}}}{2N_c T^2} + S_{\mathrm{pert}}(T) + \log(N_c)$$

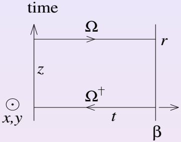




Spectral representation

- Two conjugate sources are placed in the medium at temperature T and a separation distance r generate a Free energy shift



$$e^{-\Delta F(r,T)} = \langle \mathrm{tr}_R \Omega(r) \mathrm{tr}_{\bar{R}} \Omega(0) \rangle_T$$

- Standard representation (ratio of partition functions)

$$e^{-\Delta F(r,T)} = \frac{Z_{R\otimes\bar{R}}(r,T)}{Z_0(T)} = \frac{\sum_n e^{-E_n^{R\otimes\bar{R}}(r)/T}}{\sum_n e^{-E_n^0(r)/T}}$$

- Spectral r-representation

$$e^{-\Delta F(r,T)} = \sum_n |\langle n, T | \mathrm{Tr}_R \Omega^\dagger | 0, T \rangle|^2 \, e^{-r w_n(T)}$$

- Inequalities

$$\partial_r \Delta F(r,T) \geq 0 \qquad \partial_r^2 \Delta F(r,T) \leq 0$$

Double heavy hadron spectrum and correlators

- String breaking for the $\bar{Q}Q \to \bar{B}B$ (level crossing)

$$M_{\bar{Q}Q}(r_c) = V_{Q\bar{Q}}(r_c) + m_{\bar{Q}} + m_Q = M_{\bar{B}} + M_B \,,$$

- No mixing

$$e^{-\Delta F(r,T)/T} = e^{-V_Q(r)/T} + \left(\sum_n e^{-\Delta_H^{(n)}/T} \right)^2$$

- Two modes model

$$V(r) = \begin{pmatrix} -\frac{4\alpha}{3r} + \sigma r & W(r) & \\ W(r) & 2\Delta & \\ & & \ddots \end{pmatrix} \,, \qquad W(r) = g e^{-mr}$$

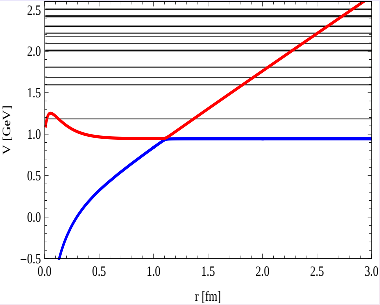


Figure: Spectrum as a function of distance and the (avoided) level crossing structure for the considered string-meson mixing scenarios. Single mixing (left panel) and multiple mixing (right panel) with RQM (c-quark).

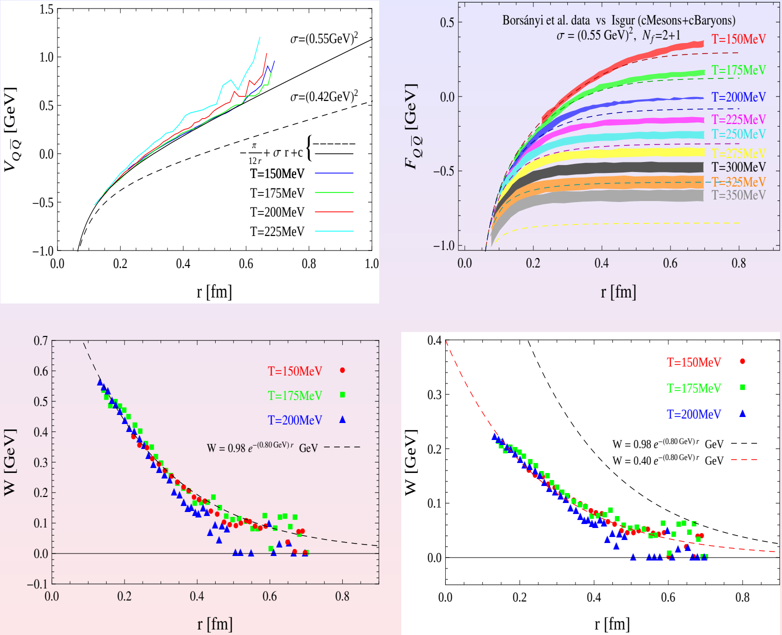


Figure: The $\bar{Q}Q \to \bar{B}B$ transition potential $W(r)$ as a function of separation

String determination

- String tension is only defined in Quenched Approximation
- String breaks. How determine string tension ?
- Using thermodynamics WITHOUT mixing

$$\sqrt{\sigma} = 0.55(14), \quad g = 0$$

- Using thermodynamics WITH mixing
  $\sqrt{\sigma} = 0.424(14), \quad g = 0.98(47), \quad m = 0.80(38)$

CONCLUSIONS

Conclusions:

- Quark Hadron Duality suggests that at low temperatures Hadrons can be considered as a complete basis of states in terms of a hadron resonance gas. The HRG works up to relatively large temperatures.
- PDG states incorporate currently just $q\bar{q}$ or qqq states which fit into the quark model. What states are needed when approaching the crossover from below ?
- Saturating at subcritical temperatures requires many hadronic states, so the excited spectrum involves relativistic effects even for heavy quarks.
- Polyakov loops in fundamental and higher representations allow to deduce multiquark quark states, gluelumps etc. containing one or several heavy quark states.