Extending a Generalized Parton Distribution from DGLAP to ERBL (Preliminary results!)

From an Overlap of Light-cone Wave-functions to a Double Distribution

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NPQCD 2016, Seville, October 18$^{\text{th}}$ 2016

Outline

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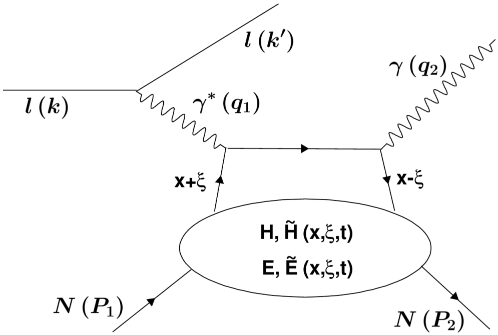
1. Introduction to Generalized Parton Distributions
   - Why Generalized Parton Distributions?
   - Definition and properties
2. Overlap and Double Distribution representations of GPDs
   - Overlap of Light-cone wave functions
   - Double Distributions
3. From an Overlap of LCWFs to a Double Distribution
   - Inversion of Incomplete Radon Transform
   - Results
4. Conclusion

Experimental access (example: DVCS)



Deeply Virtual Compton Scattering channel of photon electroproduction.

$$\Delta = P_2 - P_1 \ , \ t = \Delta^2 < 0$$

$$Q^2 = -q_1^2 > 0$$

$$P = \frac{1}{2}(P_1 + P_2) \ , \ \xi = -\frac{\Delta^+}{2\,P^+}$$

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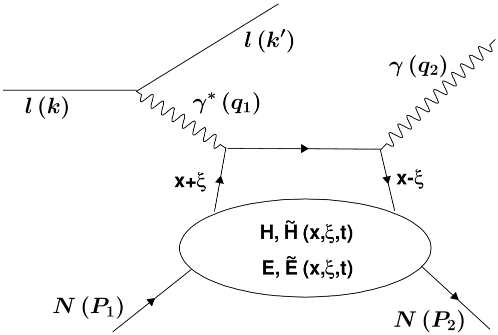

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Compton Form Factors: **(Belitsky et al., 2002)**

$$\mathcal{F}\left(\xi, t, Q^2\right) = \int_{-1}^{1} \mathrm{d}x \, C\left(x, \xi, \alpha_S\left(\mu_F\right), \frac{Q}{\mu_F}\right) F\left(x, \xi, t, \mu_F\right), \tag{1}$$

where $F \in \left\{H, E, \tilde{H}, \tilde{E}, ...\right\}$ is a Generalized Parton Distribution.

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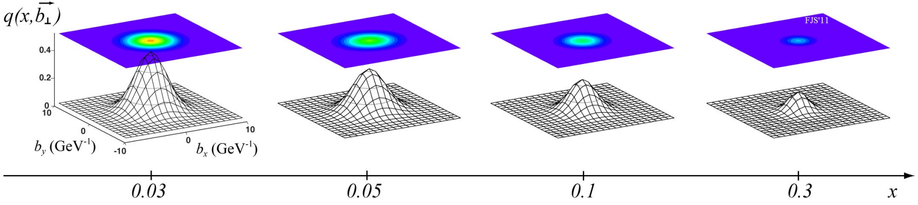

Figure: Hadron tomography.

Nucleon spin

- Ji's decomposition of the nucleon spin: **(Ji, 1997)**

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- Ji's sum rule:

$$\begin{aligned} J^q &= \frac{1}{2}\int_{-1}^{1} x\left[H^q(x,\xi,0) + E^q(x,\xi,0)\right] \mathrm{d}x \\ &= \frac{1}{2}\int_{0}^{1} x\left[q(x) + \bar{q}(x)\right] \mathrm{d}x + \frac{1}{2}\int_{-1}^{1} x\, E^q(x,0,0)\, \mathrm{d}x \\ &= \frac{1}{2}\Delta q + L^q \,. \end{aligned} \tag{4}$$

Definition of GPDs

- Quark GPD: **(Müller et al., 1994; Radyushkin, 1996; Ji, 1997)**

$$F^q(x,\xi,t) = \frac{1}{2}\int \frac{\mathrm{d}z^-}{2\pi}\, e^{i\,x\,P^+ z^-}\, \left\langle P_2 \left| \bar{q}(-z)\,\gamma^+ q(z) \right| P_1 \right\rangle \Big|_{z^+=0,\, z_\perp=0}\,, \tag{5}$$

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- Link to PDFs and Form Factors:

$$\int \mathrm{d}x\, H^q(x,\xi,t) = F_1^q(t) \quad , \quad \int \mathrm{d}x\, E^q(x,\xi,t) = F_2^q(t) \, , \tag{8}$$

$$H^q(x,0,0) = \theta(x)\, q(x) - \theta(-x)\, \bar{q}(-x) \, . \tag{9}$$

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- Positivity: **(Pire et al., 1999)**

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  - Cauchy-Schwarz theorem in Hilbert space.

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Light-cone wave functions (LCWFs)

(Brodsky and Lepage, 1989)

- A given *hadronic state* is decomposed in a **Fock basis**:

$$|H;P,\lambda\rangle = \sum_{N,\beta} \int [\mathrm{d}x]_N \left[\mathrm{d}^2\mathbf{k}_\perp\right]_N \Psi^{\lambda}_{N,\beta}\left(x_1,\mathbf{k}_{\perp 1},\ldots\right) |N,\beta;k_1,\ldots,k_N\rangle \ , \tag{12}$$

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- For example, for the pion:

$$|\pi\rangle = \sum_{q\bar{q}} \psi^\pi_{q\bar{q}} \, |q\bar{q}\rangle + \sum_{q\bar{q}g} \psi^\pi_{q\bar{q}g} \, |q\bar{q}g\rangle + \ldots$$

Overlap of LCWFs

(Diehl et al., 2001; Mezrag, 2015)

- The GPD can be then computed as an overlap of LCWFs:

$$H^q(x,\xi,t) = \sum_{N,\beta} \sqrt{1-\xi}^{\,2-N} \sqrt{1+\xi}^{\,2-N} \sum_{j=1}^{N} \delta_{s_j,q} \int [\mathrm{d}\bar{x}]_N \left[\mathrm{d}^2\bar{\mathbf{k}}_\perp\right]_N \delta(x-\bar{x}_j)\, \Psi^*_{N,\beta}(\Omega_2)\, \Psi_{N,\beta}(\Omega_1)\,, \tag{13}$$

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- DD representation of GPDs:

$$H^q(x,\xi,t) = \int_\Omega \mathrm{d}\beta\,\mathrm{d}\alpha\,\left(F^q(\beta,\alpha,t) + \xi\,G^q(\beta,\alpha,t)\right)\delta(x-\beta-\alpha\xi)\,. \quad (14)$$

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One-component DD (1CDD)

- BMKS gauge: **(Belitsky et al., 2001)**

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- Pobylitsa gauge: **(Pobylitsa, 2004)**

$$H(x,\xi,t) = (1-x) \int_\Omega \mathrm{d}\beta\, \mathrm{d}\alpha\, f_P(\beta,\alpha,t)\, \delta(x-\beta-\alpha\xi)\ . \qquad (18)$$

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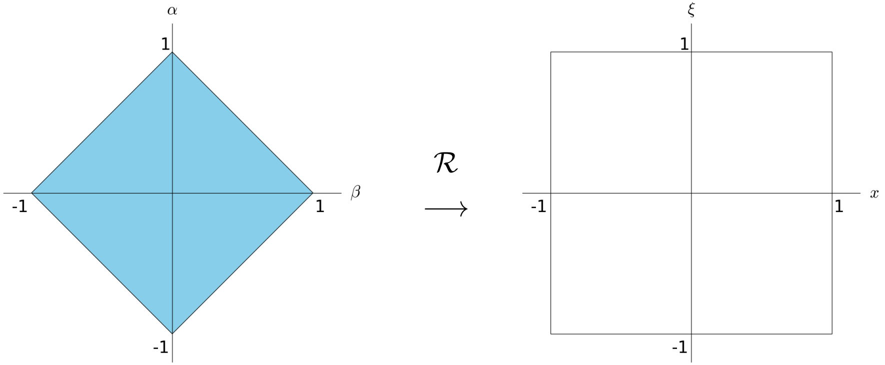
Radon transform



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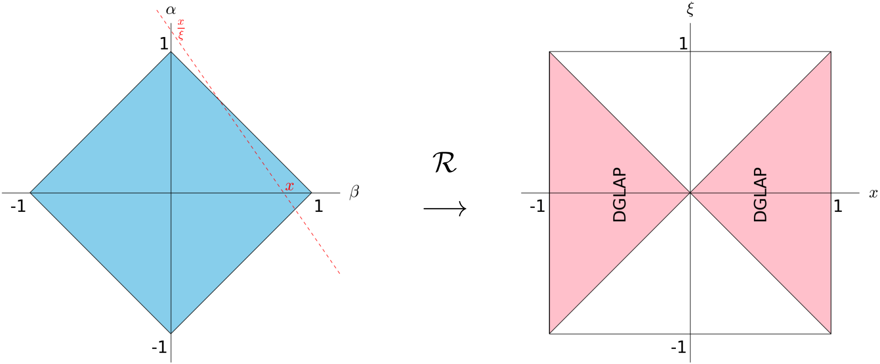

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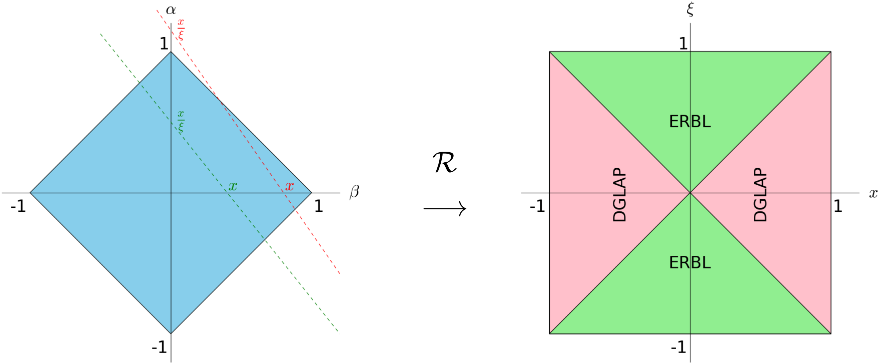

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  - ERBL region: $|x| < |\xi|$.

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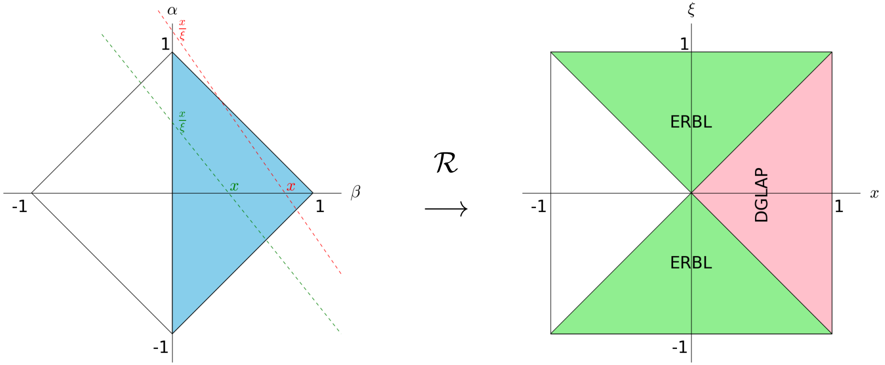
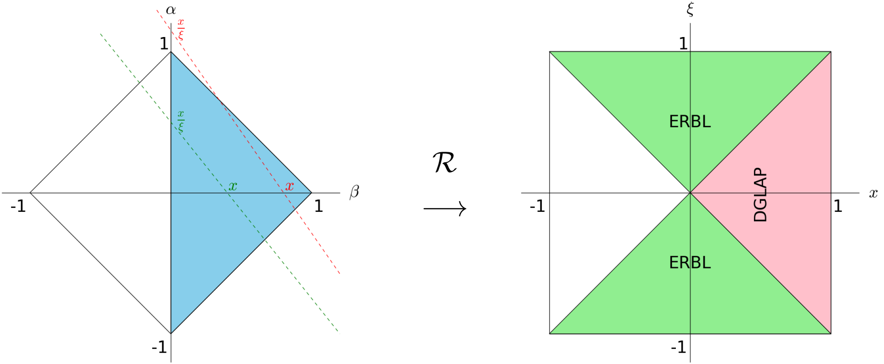
- If model fulfills Lorentz invariance:
  - DD $f(\beta, \alpha)$ **exists** (as a *distribution*) and is **unique** (if it is a *function*).
  - We can reconstruct the GPD everywhere.

(Moutarde, 2015)

Support properties

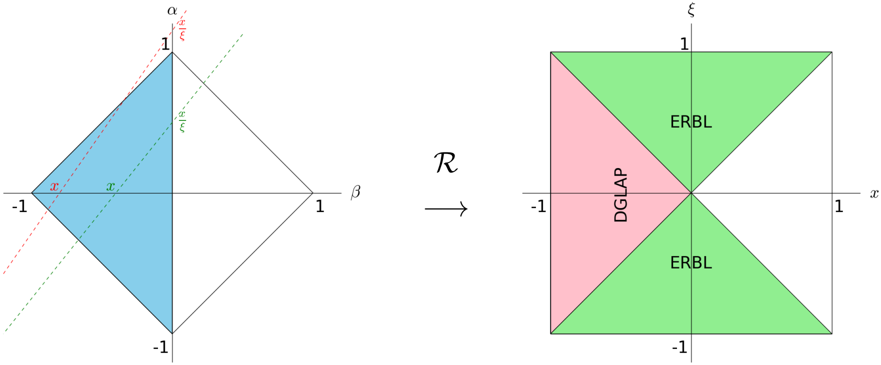


Support properties



- Valence GPD: $H(x,\xi) = 0$ for $-1 < x < -|\xi| \Longrightarrow f(\beta,\alpha) = 0$ for $\beta < 0$.

Support properties



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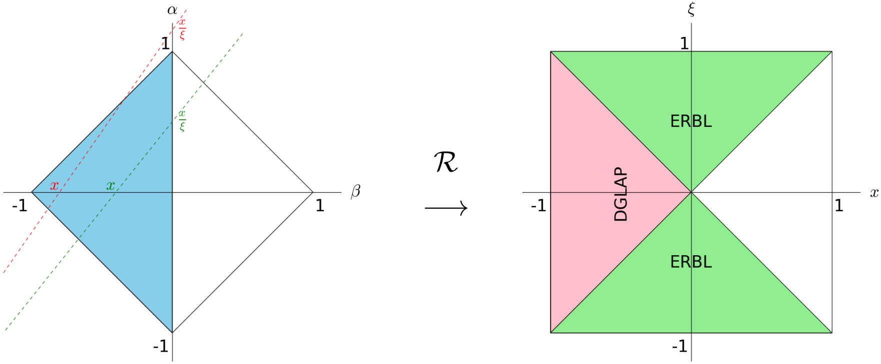

- Valence GPD: $H(x, \xi) = 0$ for $-1 < x < -|\xi| \implies f(\beta, \alpha) = 0$ for $\beta < 0$.
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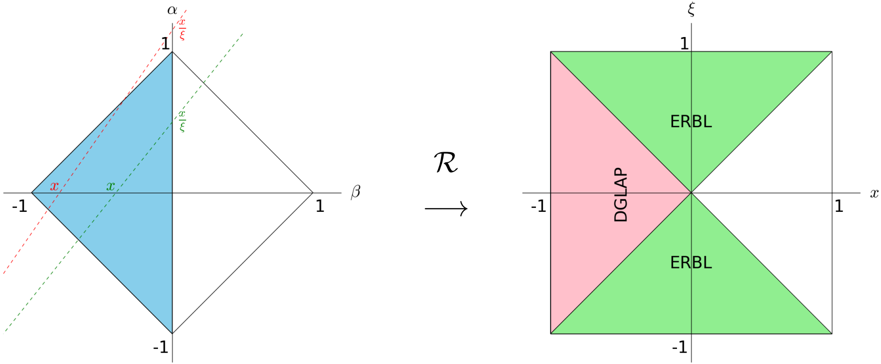
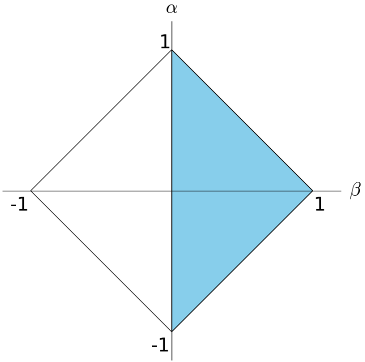
Support properties

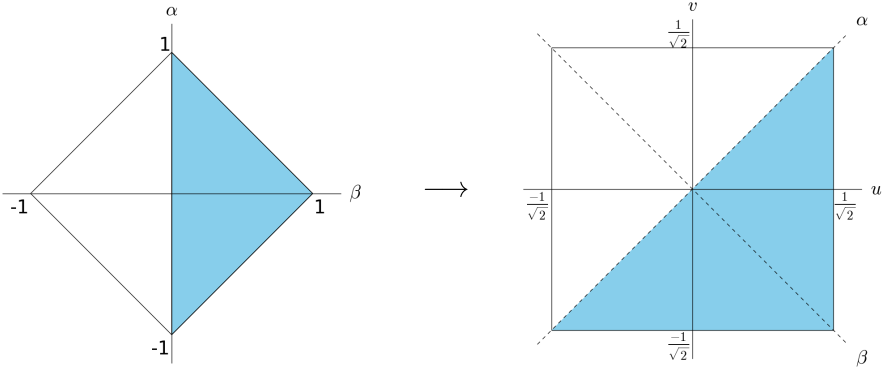


- Valence GPD: $H(x,\xi) = 0$ for $-1 < x < -|\xi| \implies f(\beta,\alpha) = 0$ for $\beta < 0$.
- Domains $\beta < 0$ and $\beta > 0$ are uncorrelated in the DGLAP region.
- Divide and conquer:
  - Better numerical stability.
  - Lesser complexity: $O(N^p + N^p) \ll O\left((N+N)^p\right)$.

Domain for the inversion



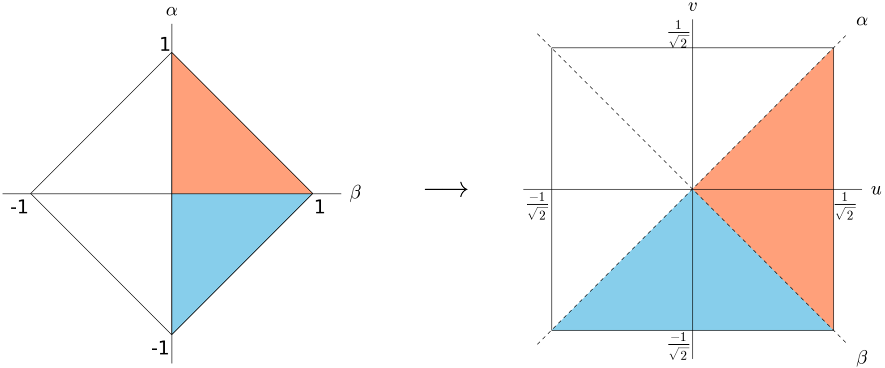
Domain for the inversion



- Rotated square $\left[-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right] \times \left[-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right]$:

$$\begin{cases} u = \dfrac{\beta + \alpha}{\sqrt{2}} & , \\ v = \dfrac{\alpha - \beta}{\sqrt{2}} & . \end{cases} \tag{21}$$

Domain for the inversion



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$$\begin{cases} u = \dfrac{\beta + \alpha}{\sqrt{2}} \quad , \\ v = \dfrac{\alpha - \beta}{\sqrt{2}} \quad . \end{cases} \tag{21}$$

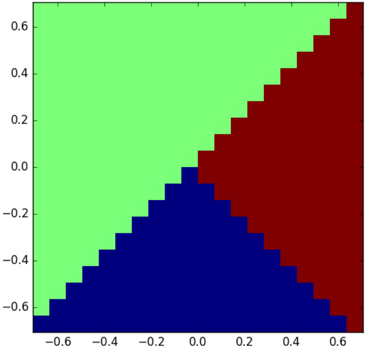
- α-parity of the DD:

$$f(\beta, -\alpha) = f(\beta, \alpha) \; . \tag{22}$$

Discretization

- Discretization of the DD (piece-wise constant):

$$\tilde{f}(u,v) = \sum_{i=0}^{N-1} \sum_{j=0}^{N-1} \tilde{f}_{ij} \, \mathbf{1}_{[u_i, u_{i+1}]}(u) \, \mathbf{1}_{[v_j, v_{j+1}]}(v) \,, \tag{23}$$



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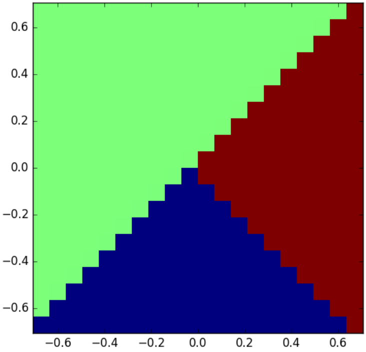
- Mesh:



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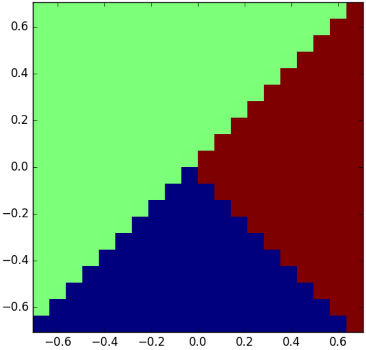
- Mesh:
  - Cells $(u,v) \to n$ columns of the matrix.



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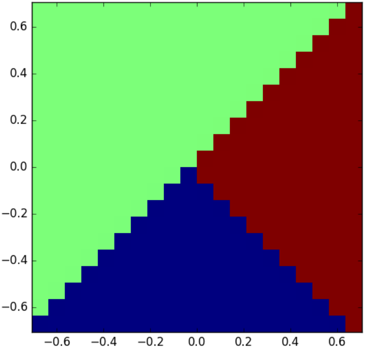
- Mesh:
  - Cells $(u,v) \rightarrow n$ columns of the matrix.
- Sampling:



Discretization

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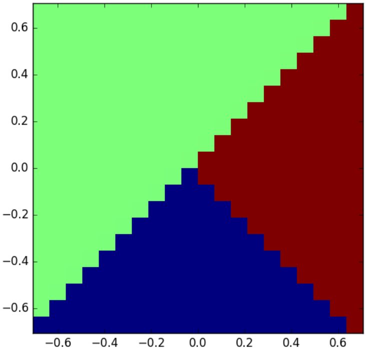
$$\tilde{f}(u,v) = \sum_{i=0}^{N-1}\sum_{j=0}^{N-1} \tilde{f}_{ij}\, \mathbf{1}_{[u_i,u_{i+\mathbf{1}}]}(u)\, \mathbf{1}_{[v_j,v_{j+\mathbf{1}}]}(v) \ , \quad (23)$$

- Mesh:
  - Cells $(u,v) \to n$ columns of the matrix.
- Sampling:
  - Random couples $(x,\xi) \to m \geq n$ lines of the matrix.



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- Mesh:
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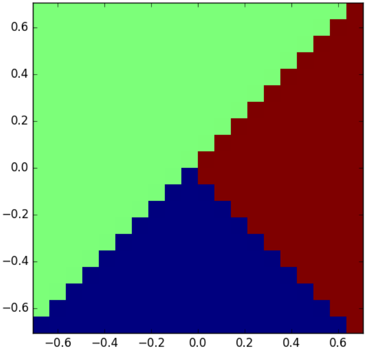
- Sampling:
  - Random couples $(x,\xi) \to m \geq n$ lines of the matrix.
- Linear problem: $A\,X = B$ where $B_k = H(x_k, \xi_k)$.



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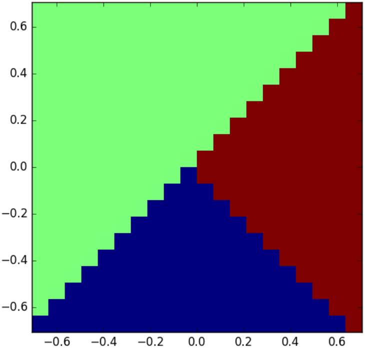
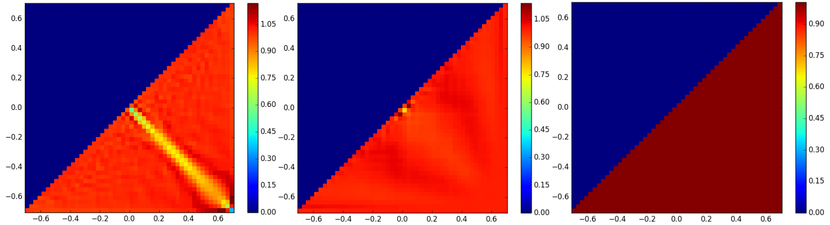


- Mesh:
  - Cells $(u,v) \rightarrow n$ columns of the matrix.
- Sampling:
  - Random couples $(x,\xi) \rightarrow m \geq n$ lines of the matrix.
- Linear problem: $A\,X = B$ where $B_k = H(x_k, \xi_k)$.
  - A full-rank: more information but also more noise.

Tests (constant DD)



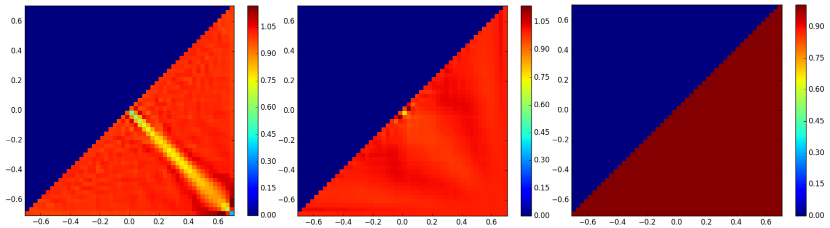
- Test with Constant DD.

$$f(\beta, \alpha) = \begin{cases} 1 & \beta > 0 \\ 0 & \beta < 0 \end{cases}$$

$$\downarrow$$

$$H(x, \xi)|_{\mathrm{DGLAP}} = \begin{cases} \frac{2x(1-x)}{1-\xi^2} & |\xi| < x < 1 \\ 0 & -1 < x < -|\xi| \end{cases}$$

Tests (constant DD)



- Test with Constant DD.
  - Goal: retrieve known DD from DGLAP GPD.

$$f(\beta,\alpha) = \begin{cases} 1 & \beta > 0 \\ 0 & \beta < 0 \end{cases}$$

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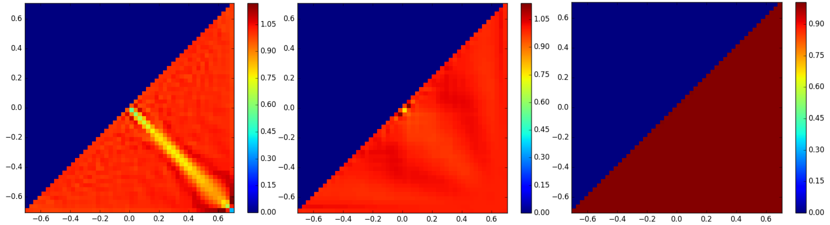

- Test with Constant DD.
  - Goal: retrieve known DD from DGLAP GPD.
- Consistent problem (discretized DD = theoretical DD):

$$f(\beta,\alpha) = \begin{cases} 1 & \beta > 0 \\ 0 & \beta < 0 \end{cases}$$

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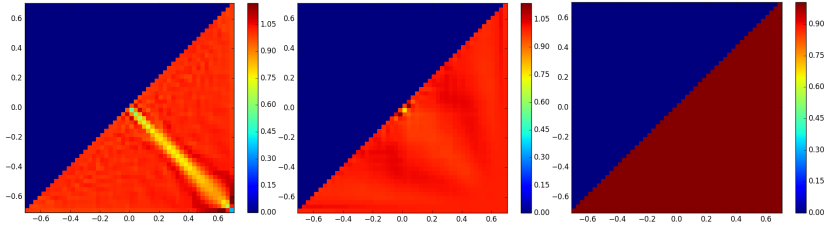


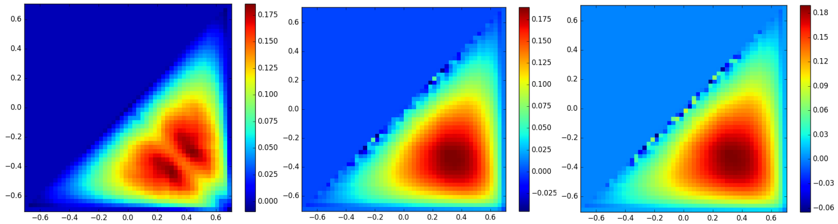
- Test with Constant DD.
  - Goal: retrieve known DD from DGLAP GPD.
- Consistent problem (discretized DD = theoretical DD):
  - Objective DD retrieved at arbitrary precision: residue decreases to 0 (machine precision).

$$f(\beta, \alpha) = \begin{cases} 1 & \beta > 0 \\ 0 & \beta < 0 \end{cases}$$

$$\downarrow$$

$$H(x, \xi)|_{\mathrm{DGLAP}} = \begin{cases} \frac{2x(1-x)}{1-\xi^2} & |\xi| < x < 1 \\ 0 & -1 < x < -|\xi| \end{cases}$$

Tests (RDDA)



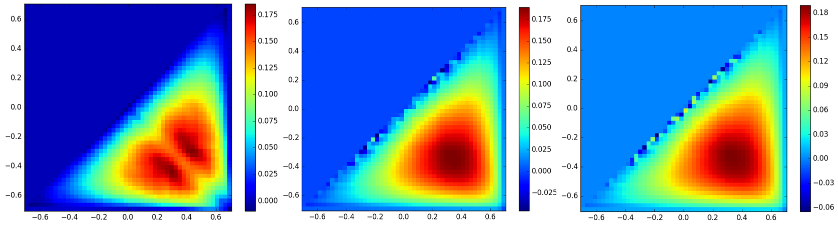
- Test with a DD inspired by Radyushkin ansatz.

$$f(\beta,\alpha) = \begin{cases} \frac{3\left(\alpha^2-(1-\beta)^2\right)\beta}{4(-1+\beta)} & \beta > 0 \\ 0 & \beta < 0 \end{cases}$$

$$\downarrow$$

$$H(x,\xi)|_{x>|\xi|} = \frac{(1-x)^3\left(3\xi+(2x-5)\xi^3-3\left(1-\xi^2\right)^2\arctan\xi\right)}{2\xi^3(1-\xi^2)^2}$$

Tests (RDDA)



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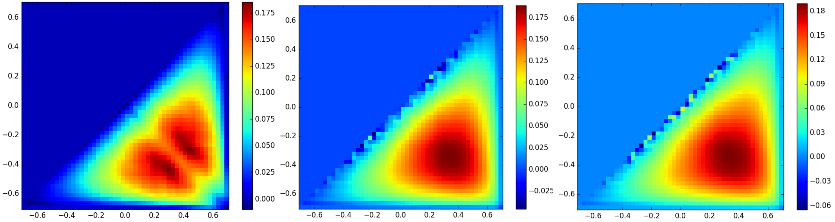

- Test with a DD inspired by Radyushkin ansatz.
  - Goal: retrieve known DD from DGLAP GPD.
- Smooth function, vanishes at the boundaries.

$$f(\beta,\alpha) = \begin{cases} \frac{3\left(\alpha^2-(1-\beta)^2\right)\beta}{4(-1+\beta)} & \beta > 0 \\ 0 & \beta < 0 \end{cases}$$

$$\downarrow$$

$$H(x,\xi)|_{x>|\xi|} = \frac{(1-x)^3\left(3\xi+(2x-5)\xi^3-3\left(1-\xi^2\right)^2\arctan\xi\right)}{2\xi^3(1-\xi^2)^2}$$

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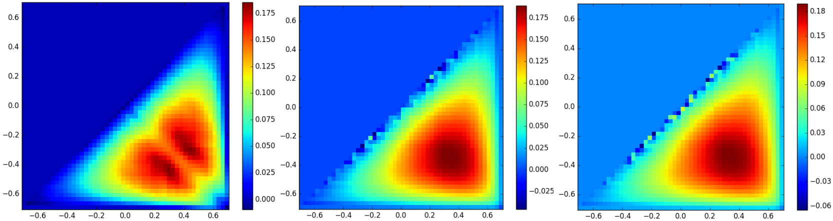

- Test with a DD inspired by Radyushkin ansatz.
  - Goal: retrieve known DD from DGLAP GPD.
- Smooth function, vanishes at the boundaries.
  - Least-squares problem: residue has a finite limit.

$$f(\beta,\alpha) = \begin{cases} \frac{3\left(\alpha^2-(1-\beta)^2\right)\beta}{4(-1+\beta)} & \beta > 0 \\ 0 & \beta < 0 \end{cases}$$

$$\downarrow$$

$$H(x,\xi)|_{x>|\xi|} = \frac{(1-x)^3\left(3\xi+(2x-5)\xi^3-3\left(1-\xi^2\right)^2\arctan\xi\right)}{2\xi^3(1-\xi^2)^2}$$

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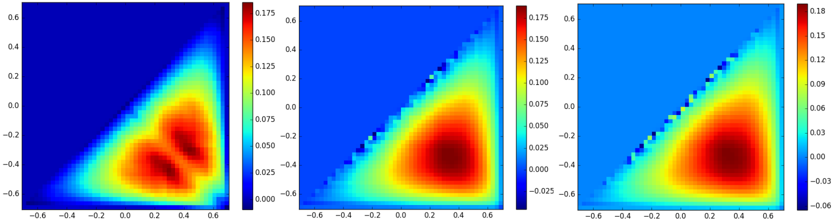


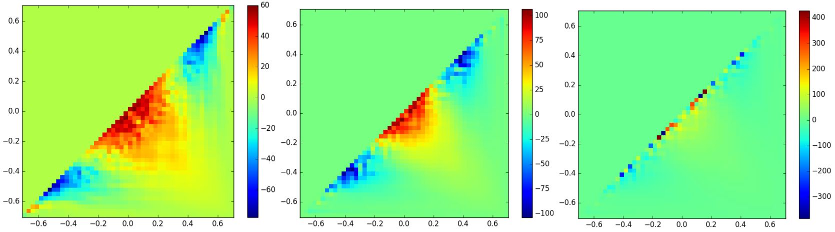
- Test with a DD inspired by Radyushkin ansatz.
  - Goal: retrieve known DD from DGLAP GPD.
- Smooth function, vanishes at the boundaries.
  - Least-squares problem: residue has a finite limit.
  - Compromise between noise on $\beta = 0$ and artifact on $\alpha = 0$.

$$f(\beta, \alpha) = \begin{cases} \frac{3\left(\alpha^2 - (1-\beta)^2\right)\beta}{4(-1+\beta)} & \beta > 0 \\ 0 & \beta < 0 \end{cases}$$

$$\downarrow$$

$$H(x, \xi)|_{x>|\xi|} = \frac{(1-x)^3 \left(3\xi + (2x-5)\,\xi^3 - 3\left(1-\xi^2\right)^2 \arctan \xi\right)}{2\xi^3\,(1-\xi^2)^2}$$

First result



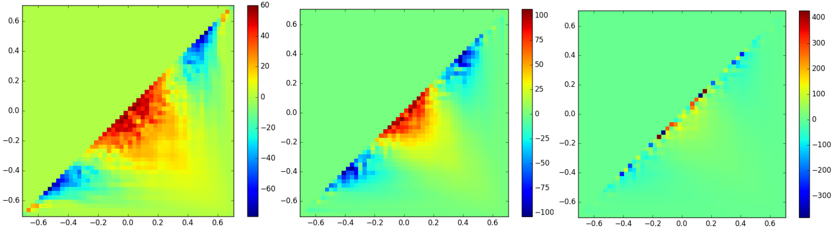
- Real application to an algebraic DSE overlap model.

$$f(\beta,\alpha) = \begin{cases} ? & \beta > 0 \\ 0 & \beta < 0 \end{cases}$$

$$\downarrow$$

$$H(x,\xi)|_{x>|\xi|} = 30\frac{(1-x)^2\left(x^2-\xi^2\right)}{\left(1-\xi^2\right)^2}$$

First result



- Real application to an algebraic DSE overlap model.
  - Goal: extend the DGLAP GPD of Ref. (Mezrag, 2015; Mezrag et al., 2016).

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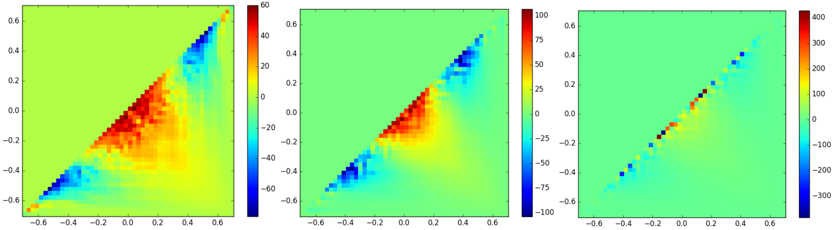
$$H(x,\xi)\big|_{x>|\xi|} = 30\frac{(1-x)^2\left(x^2-\xi^2\right)}{\left(1-\xi^2\right)^2}$$

First result



- Real application to an algebraic DSE overlap model.
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- Singularities in BMKS gauge:

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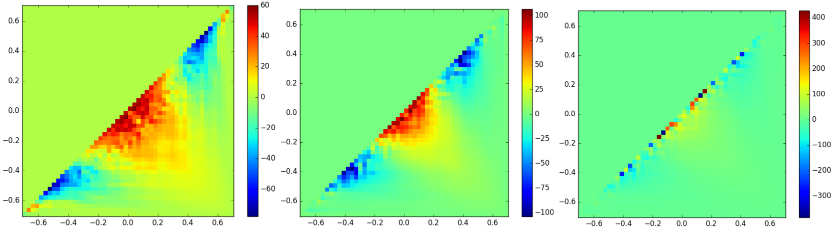

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  - Physical or numerical?

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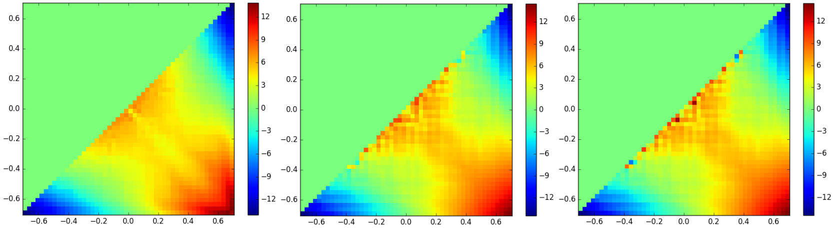
First result



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- Smooth function in Pobylitsa gauge:

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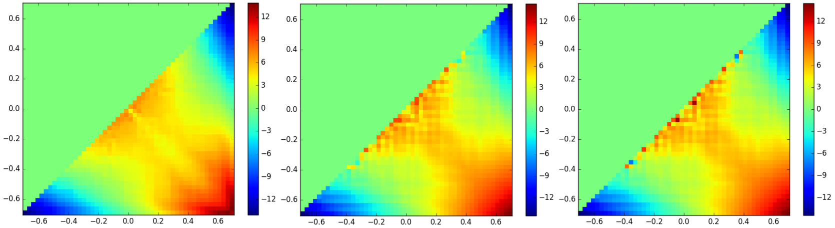
First result



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- Smooth function in Pobylitsa gauge:
  - $(1-x)^2$ behavior of the GPD.

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$$\downarrow$$

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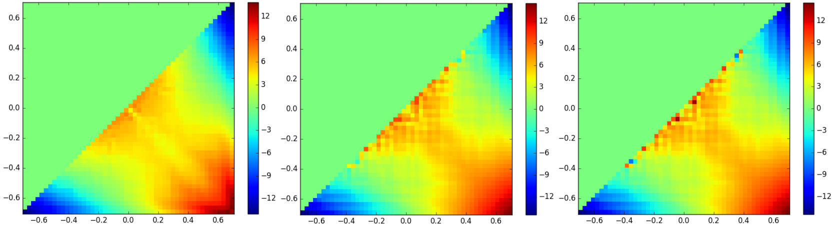
First result



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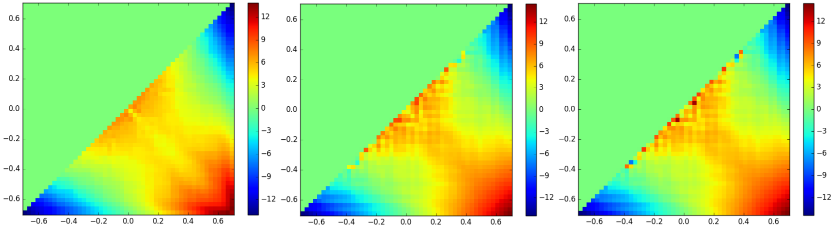
- Smooth function in Pobylitsa gauge:
  - $(1-x)^2$ behavior of the GPD.
  - Positivity of the Pobylitsa gauge.

$$f(\beta,\alpha) = \begin{cases} ? & \beta > 0 \\ 0 & \beta < 0 \end{cases}$$

$$\downarrow$$

$$H(x,\xi)|_{x>|\xi|} = 30\frac{(1-x)^2\left(x^2-\xi^2\right)}{\left(1-\xi^2\right)^2}$$

First result



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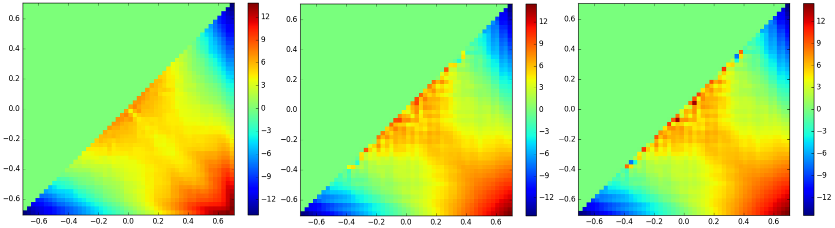
  - $(1-x)^2$ behavior of the GPD.
  - Positivity of the Pobylitsa gauge.

$$f(\beta,\alpha) = \begin{cases} \frac{30}{4}\left(1 - 3\alpha^2 - 2\beta + 3\beta^2\right) & \beta > 0 \\ 0 & \beta < 0 \end{cases}$$

$$\downarrow$$

$$H(x,\xi)|_{x>|\xi|} = 30\frac{(1-x)^2\left(x^2-\xi^2\right)}{\left(1-\xi^2\right)^2}$$

First result



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Quantitative comparison of DDs

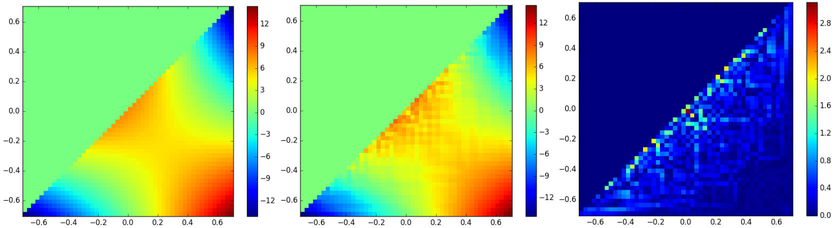

Figure: Quantitative comparison for the Overlap results. Left: Theoretical discretized DD. Middle: Numerical solution at tolerance 10^{-6}. Right: Absolute difference.

$$f(\beta,\alpha) = \begin{cases} \frac{3\left(\alpha^2-(1-\beta)^2\right)\beta}{4(-1+\beta)} & \beta > 0 \\ 0 & \beta < 0 \end{cases}$$

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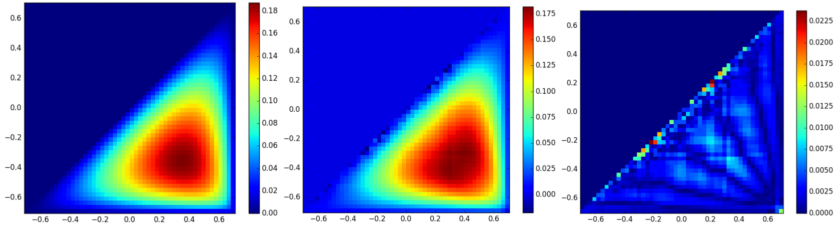

Figure: Quantitative comparison for the RDDA results. Left: Theoretical discretized DD. Middle: Numerical solution at tolerance 10^{-6}. Right: Absolute difference.

$$f(\beta, \alpha) = \begin{cases} \frac{30}{4}\left(1 - 3\alpha^2 - 2\beta + 3\beta^2\right) & \beta > 0 \\ 0 & \beta < 0 \end{cases}$$

Quantitative comparison of GPDs

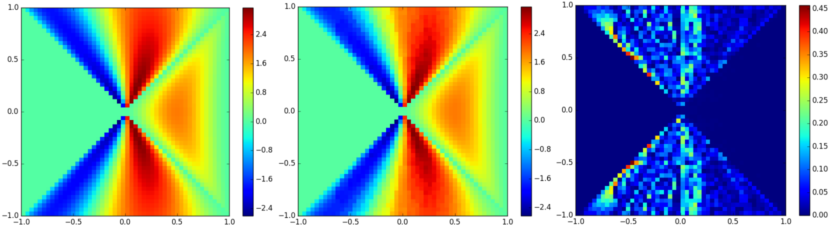

Figure: Quantitative comparison for the Overlap GPD obtained from the numerical DD solution. Left: Theoretical GPD. Middle: Numerical solution at tolerance 10^{-6}. Right: Absolute difference.

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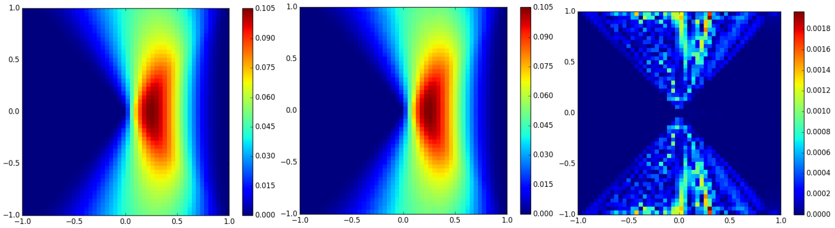

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$$H\left(x,\xi\right)\big|_{x>|\xi|} = \frac{\left(1-x\right)^3\left(3\xi+\left(2x-5\right)\xi^3-3\left(1-\xi^2\right)^2\arctan\xi\right)}{2\xi^3\left(1-\xi^2\right)^2}$$

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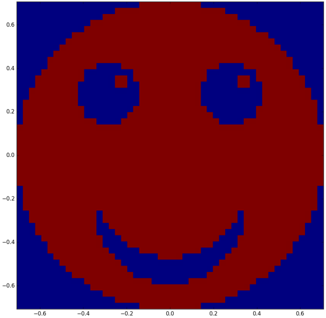

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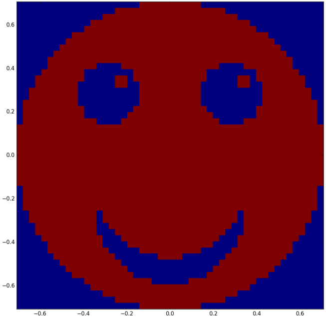

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DSE Overlap DD

Recall the GPD: **(Mezrag, 2015; Mezrag et al., 2016)**

$$H(x,\xi) = 30\,\frac{(1-x)^2\left(x^2-\xi^2\right)}{\left(1-\xi^2\right)^2}\,. \tag{24}$$

After trial and error (to reproduce the numerical result in Pobylitsa gauge that looks like a polynomial):

$$f_P(\beta,\alpha) = \frac{30}{4}\left(1-3\alpha^2-2\beta+3\beta^2\right)\,. \tag{25}$$

And with the mapping from Ref. **(Mueller, 2014)** (Eq. (10) of the article), we get the singular indeed DD in BMKS gauge:

$$\begin{aligned} f_M(\beta,\alpha) = \frac{30}{4}\Bigg(&\alpha^2\left(3-\frac{3}{(|\alpha|+\beta)^4}\right)+\beta^2\left(\frac{3}{(|\alpha|+\beta)^3}-3\right)\\ &+\beta\left(4-\frac{4}{(|\alpha|+\beta)^3}\right)+\frac{2}{(|\alpha|+\beta)^2}-2\Bigg)\\ &+\frac{30}{4}\left(1-\alpha^2\right)\delta(\beta)\,. \end{aligned} \tag{26}$$